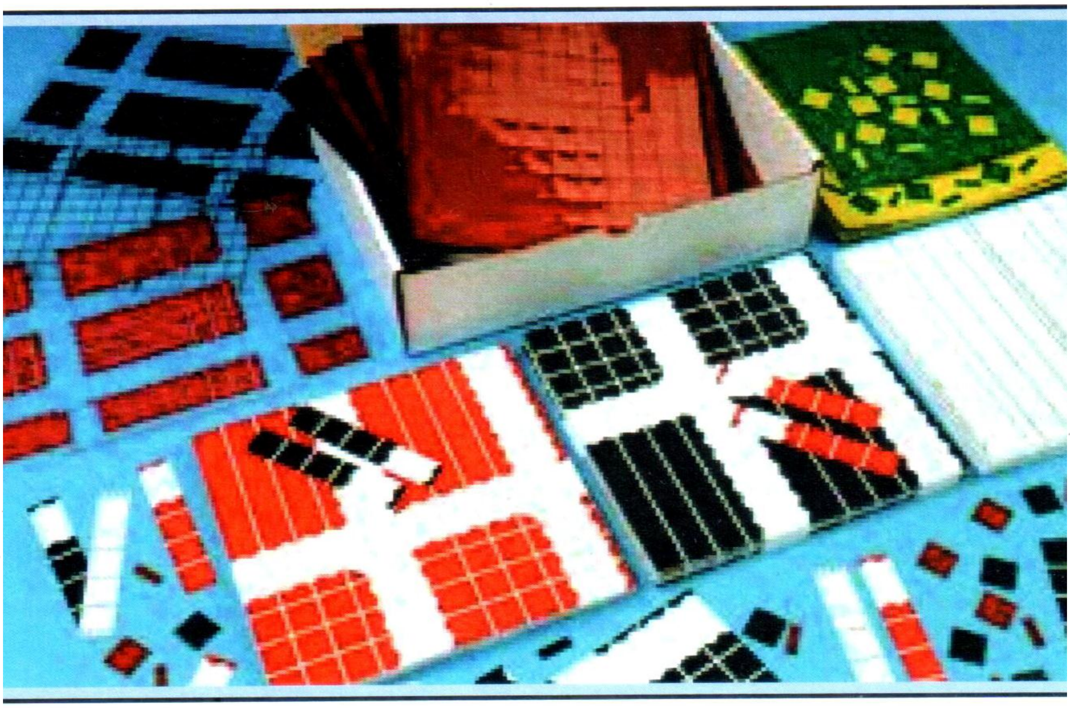
DEPARTMENT OF EDUCATION

GRADE 8

MATHEMATICS

STRAND 6



PATTERNS AND ALGEBRA

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SECRETARY'S MESSAGE

Achieving a better future by individual students and their families, communities or the nation as a whole, depends on the kind of curriculum and the way it is delivered.

This course is part and parcel of the new reformed curriculum. The learning outcomes are student-centered with demonstrations and activities that can be assessed.

It maintains the rationale, goals, aims and principles of the national curriculum and identifies the knowledge, skills, attitudes and values that students should achieve.

This is a provision by Flexible, Open and Distance Education as an alternative pathway of formal education.

The course promotes Papua New Guinea values and beliefs which are found in our Constitution and Government Policies. It is developed in line with the National Education Plans and addresses an increase in the number of school leavers as a result of lack of access to secondary and higher educational institutions.

Flexible, Open and Distance Education curriculum is guided by the Department of Education‟s Mission which is fivefold:

- to facilitate and promote the integral development of every individual
- to develop and encourage an education system that satisfies the requirements of Papua New Guinea and its people
- to establish, preserve and improve standards of education throughout Papua New Guinea
- to make the benefits of such education available as widely as possible to all of the people
- to make the education accessible to the poor and physically, mentally and socially handicapped as well as to those who are educationally disadvantaged.

The college is enhanced through this course to provide alternative and comparable pathways for students and adults to complete their education through a one system, two pathways and same outcomes.

It is our vision that Papua New Guineans‟ harness all appropriate and affordable technologies to pursue this program.

I commend all the teachers, curriculum writers and instructional designers who have contributed towards the development of this course .

DR. UKE KOMBRA, PhD
Acting Secretary for Education

STRAND 6: PATTERNS AND ALGEBRA

Introduction

Dear Student,

This is the sixth Strand of the Grade 8 mathematics Course.

This Strand consists of four Sub-strands:

Sub-strand 1: Algebraic Expressions
Sub-strand 2: Indices
Sub-strand 3: Polynomials
Sub-strand 4: Mathematical Equations

Sub-strand 1 – **Algebraic Expressions** – You will learn to recognise and use patterns in processes and manipulate algebraic expressions in solving problems in real life situations.

Sub-strand 2 – **Indices** – You will learn the rules and how to simplify positive, zero and negative indices.

Sub-strand 3 – **Polynomials** – You will learn to classify polynomials, perform operations on simple polynomials, identify like and unlike polynomials, factors, common factors and apply the laws of exponents and the Distributive law of algebra.

Sub-strand 4 – **Mathematical Equations** – You will learn to manipulate algebraic expressions and solve equations by the substitution method and other algebraic problem solving techniques to solve simple equations.

You will find that each lesson has reading materials to study, worked examples to help you, and a Practice Exercise. The answers to practice exercises are given at the end of each sub-strand.

All the lessons are written in simple language with comic characters to guide you and many worked examples to help you. The practice exercises will help you to learn the process of working out problems.

We hope you enjoy using this Stand.

All the best!

Mathematics Department
FODE

STUDY GUIDE

Follow the steps given below as you work through the Strand.

Step 1: Start with SUB-STRAND 1 Lesson 1 and work through it.

Step 2: When you complete Lesson 1, do Practice Exercise 1.

Step 3: After you have completed Practice Exercise 1, check your work. The answers are given at the end of SUB-STRAND 1.

Step 4: Then, revise Lesson 1 and correct your mistakes, if any.

Step 5: When you have completed all these steps, tick the check-box for the Lesson, on the Contents Page (page 3) like this:

[√] Lesson 1: Translating Algebraic Phrases to Algebraic Expressions

Then go on to the next Lesson. Repeat the process until you complete all of the lessons in Sub-strand 1.

> As you complete each lesson, tick the check-box for that lesson, on the Content"s page 3, like this [√].This helps you to check on your progress.

Step 6: Revise the Sub-strand using Sub-strand 1 Summary, then do Sub-strand test 1 in Assignment 6.

Then go on to the next Sub-strand. Repeat the process until you complete all of the four Sub-strands in Strand 6.

Assignment: (Four Sub-strand Tests and a Strand Test)
When you have revised each Sub-strand using the Sub-strand Summary, do the Sub-strand Test for that Sub-strand in your Assignment book. The Strand book tells you when to do each Sub-strand Test.

When you have completed the four Sub-strand Tests, revise well and do the Strand test. The Workbook tells you when to do the Strand Test.

> The Sub-strand Tests and the Strand test in the Assignment will be marked by your Distance Teacher. The marks you score in each Assignment will count towards your final mark. If you score less than 50%, you will repeat that Assignment.

Remember, if you score less than 50% in three Assignments, your enrolment will be cancelled. So, work carefully and make sure that you pass all of the Assignments.

SUB-STRAND 1

ALGEBRAIC EXPRESSIONS

SUB - STRAND 1: ALGEBRAIC EXPRESSIONS

Introduction

Everybody knows that language is a very important tool in communication. We use it to express our feelings and to transmit our ideas to others.

In Mathematics, we also need language to explore our knowledge to communicate. That is how Algebra becomes important in the world of Mathematics.

Algebra is an Arabic word meaning bringing together broken parts. It allows us to find unknown numbers from specified information.

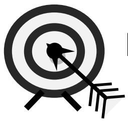
Algebra is all around us. It is constantly used by engineers, scientists, economists and by people who work with computers in banking and in many industries. It has many practical solutions too. For example, whenever we use a formula to figure out the amount of profit we will earn from a certain business, we use algebra.

In this Sub strand, you will:

- define and work with algebraic expressions
- create record, analyse and generalize number patterns using words and algebraic symbols in a variety of ways
- apply process patterns in problem solving
- simplify and solve algebraic expressions.

Lesson 1: Translating Mathematical Phrases to Algebraic Expressions

Most of the problems in real life can be solved by using algebraic processes. It is therefore important that we carefully study this lesson.



In this lesson, you will:

- define the terms variable and constant
- differentiate algebraic expression from numerical expressions
- translate words into algebraic symbols.

The ability to use symbols to represent mathematical phrases is one important skill in mathematics that we learn. We can do these by using numbers, symbols of operations such as +, –, x or ÷, and the grouping symbols like parentheses ( ), brackets [ ], or braces { }, and variables.

You learnt about variables in your Grade 7 Mathematics. If we take any letter of the English alphabet

- **A variable is a letter, a box, or a blank which takes the place of a number. It is a symbol of a number we do not know yet. It is usually a letter like x or y.**
- **A constant is a number with a fixed value, that is, a value that does not change.**

Some examples we looked at in grade 7 were:

1) $\square + 34$, the variable is $\square$ and the constant is 34.

2) $\frac{13}{x}$, the variable is x and the constant is 13.

3) 3 + _____ + 4, the variable is _____ and the constants are 3 and 4.

Mathematical Expressions convey ideas or values. Expressions can be a single term or a number of terms separated by a minus (–) or a plus (+) sign. There are two different types of mathematical expressions. These are:

1) Numerical expressions
2) Algebraic expressions

- **Numerical expressions are made up of numbers joined by signs of operations and grouping symbols.**
- **Algebraic expressions consist of numbers and one or more variables, symbols and sometimes constants.**

In Grade 7 we learnt to do away with the use of the letter **x** as multiplication signs in Algebra. We used dots (•) or parentheses ( )( ) instead. We will continue to use dots or parentheses in expressions.

Examples of numerical expressions

1) $13 + (3 - 7)$
2) $5 - 2(3)$
3) $6(4) + 2(8)$

Examples of algebraic expressions

1) $5c - (2c + 3)$
2) $2(x + 2)$
3) $4s + s - 3n$
4) $3n - 5$
5) x^2y^3
6) $2x^2 + x + 1$

In example 4 above the number 3 in front of **n** is called the coefficient of the variable.

$$\mathbf{3n - 5}$$

Coefficient

Constant

Variable

In algebraic expressions, it is not necessary to write the symbol for multiplication. The product of **x** and **y** can be written simply as **xy**.

In cases where the expression contains a number and a variable, the number always precedes the variable.

Example The product of 7 and n is written as 7n.

Translating Phrases to Mathematical Symbols

Most mathematical word problems we face can be solved easily when the problem is written as a mathematical expression and in symbols. How can we write English words or phrases as mathematical expressions?

There are English words that mean one of the four operations or equal.

Example “sum” means to add

“product” means multiply

There are many other words that can be used to mean any of the four operations.

Here are some mathematical translations of English words.

Addition(+)	**Subtraction (-)**	**Multiplication (x)**	**Division(÷)**	**Equals(=)**
Sum Add Put together Altogether Increased by More than	Take away Subtract Decreased by Difference of Less than Difference between Fewer than	Product of Of Times Twice Multiplied by	Per Out of Quotient Divide by Share Ratio of	Is Are Was Were Will be Gives Yields

Knowing the key words above makes it a lot easier in translating English to Mathematical expressions.

Example 1

The Sum of ten and twelve can be written as 10 + 12 in numbers and signs.

Example 2

The product of eight and six take away 4 is written as 8(6) – 4

Example 3

The sum of n and three is written as n + 3

Here is a table showing a few more English sentences changed to mathematical expressions.

Phrase	**Mathematical expression**
Increase five by two	5 + 2
Three more than a number	n + 3 (n stands for a number)
Twice four	2 × 4
A quarter of twenty kina	$\frac{1}{4}$ × K 20.
Share twenty kina between two students	20 ÷ 2
Difference of 20 and 10	20 – 10
Decrease six by a number	6 – n
The sum of a and b divided by two is	$\frac{a + b}{2} =$
The product of 2 and a number	2(x)
Sum of 5 and the product of 2 and 4	5 + 2(4)

Sometimes we can express in our own words what an algebraic expression can mean.

Example 1 The algebraic expression b – 4 can mean the following:

Four less than a number
A number take away 4
The difference of a number and 4
Decrease a number by four
Subtract four from a number

Example 2 The algebraic expression 2(x + 2)

Two times the sum of a number and 2
Sum of a number and 2 multiplied by 2
The product of 2 and the sum of a number and 2

You must learn the key terms well, to make it easier when translating to mathematical expressions. You must do a lot of practice to help you remember.

NOW DO PRACTICE EXERCISE 1

Practice Exercise 1

1. State the coefficient of **x** in the following:

   a) 4x b) 7x c) 5 + 6x

2. What is the constant term of 4y + 6.

3. Write as algebraic expression each of the following.

   a) The sum of x and 3.
   b) Decrease z by 5.
   c) Twice a.
   d) Increase x by 4.
   e) Divide z by 4.
   f) The product of a, b and c.

4. Match the expressions on the left with the correct answer on the right.

a)	Add 2 to 5x.	(i)	ay
b)	Product of 2x and 4.	(ii)	3y – 2
c)	Decrease 3y by 2.	(iii)	x + y + 3
d)	The sum of x, y and 3	(iv)	5x + 2
e)	The product of a and y	(v)	4(2x)

5. Write the expression **y + 3** in words.

6. Indicate which of the following is an algebraic expression or a numerical expression.

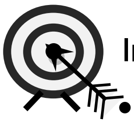
   a) 13 + 3(2 + 5)

   b) 3(x + 2) – 6

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUB- STRAND 1

Lesson 2: Evaluating Algebraic Expressions

In the last lesson we identified variables and constants in expressions and translated mathematical sentences into algebraic expressions.



In this lesson, you will:

- evaluate and simplify algebraic expressions

In grade 7, you learnt how to evaluate numerical and algebraic expressions. You learnt that to evaluate means to find the value of an expression.

Here are some examples of evaluating numerical expressions you studied in grade 7.

1) $3 + (4 \times 2) = 3 + 8$
$= 11$

2) $2(4) = 8$

3) What is $10 + 4$? Answer: 14

4) Evaluate $10 + 4$ Answer: 14

Before we can evaluate numerical or algebraic expressions we must learn the order of operations. The order of operations is a standard order used when conducting arithmetic operations, which were devised by mathematicians and are still in used today. It is used for calculations involving more than one arithmetic operation.

Order of operation

Rule 1: Perform any calculations inside parentheses.
Rule 2: Next perform all multiplication and division, working from the left to the right.
Rule 3: Lastly perform all addition and subtraction, working from the left to the right.

Let us now use the rules to evaluate example 1 above.

$3 + (4 \times 2) = 3 + \mathbf{(4 \times 2)}$ parentheses
$= 3 + 8$ addition
$= 11$

Turn to the next page and study more examples.

Example 2

16 ÷ 8 – 2 = **16 ÷ 8** – 2	Division
= 2 – 2	Subtraction
= 0	

Example 3

3(25 – 11) = 3**(25 – 11)**	Parentheses
= 3(14)	Multiplication
= 42	

The following examples involve more than two operations.

Example 4

Evaluate 3 + 6 x (5 + 4) ÷ 3 - 7 using the order of operations.

Solution	3 + 6 x **(5 + 4)** ÷ 3 – 7	Parentheses
	= 3 + **6 x 9** ÷ 3 – 7	Multiplication
	= 3 + **54 ÷ 3** – 7	Division
	= **3 + 18** – 7	Addition
	= 21 – 7	Subtraction
	= 14	

Example 5

Evaluate 9 - 5 ÷ (8 - 3) x 2 + 6

Solution	9 - 5 ÷ **(8 - 3)** x 2 + 6	Parentheses
	= 9 – **5 ÷ 5** x 2 + 6	Division
	= 9 – **1 x 2** + 6	Multiplication
	= **9 – 2** + 6	Subtraction
	= 7 + 6	Addition
	= 13	

In examples 4 and 5 the multiplication and division operations were evaluated from left to right according to rule 2 while addition and subtraction were evaluated from left to right according to rule 3.

Just like evaluating numerical expressions, order of operations must be followed in evaluating algebraic expressions.

How do we evaluate algebraic expressions?

To evaluate an algebraic expression means to find the value of the expression. This is done by replacing the variable in the expression by a given number.

Example 1

Evaluate $3y + 2y$ when $y = 5$

Solution: Step 1 Replace each letter with its assigned value

$3(5) + 2(5)$

Step 2 Perform the operations following the order of operations.

$\mathbf{3(5)} + 2(5)$

$= 15 + \mathbf{2(5)}$

$= 15 + 10$

$= 25$

Example 2

Evaluate $5r + 3x + 2e$ when $r = 4$, $e = 5$ and $x = 6$

Solution: Step 1 Replace each letter with its assigned value

$5(4) + 3(6) + 2(5)$

Step 2 Perform the operations following the order of operations

$5(4) + 3(6) + 2(5)$

$= 20 + 18 + 10$

$= 48$

In the next set of examples we will evaluate the expressions without writing the steps.

Example 3 Find the value of $6(3x – 5)$ given $x = 2$

$6(3x – 5) = 6[(3x\ 2) – 5]$

$= 6[6 – 5]$

$= 6[1]$

$= 6$

Example 4 Evaluate $n^3 – n^2$ given $n = 2$

$n^3 – n^2 = 2^3 – 2^2$

$= 8 – 4$

$= 4$

Example 5 Evaluate $3x^3 + 2x^2 + x + 1$ given $x = 1$.

$$\begin{aligned} 3x^3 + 2x^2 + x + 1 &= 3(1)^3 + 2(1)^2 + 1 + 1 \\ &= 3 + 2 + 1 + 1 \\ &= 7 \end{aligned}$$

Example 6 If $a = 5$, $b = 6$ and $c = 3$, evaluate $\frac{ab}{c}$.

$$\begin{aligned} \frac{ab}{c} &= \frac{5(6)}{3} \\ &= \frac{30}{3} \\ &= 10 \end{aligned}$$

Example 7 If $b= 6$, $c = 3$ evaluate $\frac{b+c}{c}$.

$$\begin{aligned} \frac{b+c}{c} &= \frac{6+3}{3} \\ &= \frac{9}{3} \\ &= 3 \end{aligned}$$

Example 8 Given $a = 3$, $b = 5$ evaluate $\frac{ab}{a}$.

$$\begin{aligned} \frac{ab}{a} &= \frac{3(5)}{3} \\ &= \frac{15}{3} \\ &= 5 \end{aligned}$$

Sometimes algebraic expressions can be simplified before finding the value.

What is simplifying?

To simplify means to make an expression simple in the sense that the expression has fewer operation signs, parentheses or other grouping symbols.

Example 1 Evaluate $4x + 3x – x$ when $x = 2$.

Steps 1 Simplify the expression by collecting like terms.

$$\begin{aligned} 4x + 3x - x &= 7x - x \\ &= 6x \quad \text{simplified form} \end{aligned}$$

Steps 2 Evaluate the simplified form.

$6x = 6(2)$
$= 12$

Example 2 Given $a = (-1)$ and $c = 2$ evaluate $7a – (4a + c)$.

Step 1 Simplify the expression by collecting like terms.

$7a – (4a + c)$
$= 7a – 4a – c$
$= 3a – c$ Simplified form

Step 2 Evaluate the simplified form.

$3a – c = 3(-1) – 2$
$= -3 – 2$
$= - 5$

The next example is the same as example 8 on page 17. See how simplifying makes it easier before evaluating expressions.

Example 3 Given $a = 3$, $b = 5$ evaluate $\frac{ab}{a}$

$\frac{\cancel{a}b}{\cancel{a}} = b$

Therefore $b = 5$

Example 4 Find the value of $6x + 2y – 3x + 4y – 3$ when $x = 3$ and $y = 2$.

Step1 Simplify the expression.

$6x + 2y – 3x + 4y – 3$

$=(6x – 3x) +(2y + 4y) – 3$

$= 3x + 6y – 3$

Step 2 Evaluate the simplified form.

$3x + 6y – 3$

$= (3 \times 3) + (6 \times 2) – 3$

$= 9 + 12 – 3$

$=18$

Be very neat with your calculations. Many algebraic problems are usually missed because students misread what is written, did not line up the columns correctly for division and subtraction or did not follow the order of operations. Always double check your answer.

Here are some more examples.

1) Evaluate $6x$ when $x = -3$

Step 1 Replace each letter with its assigned value

$6(-3)$

Step 2 Perform the operation to find the value of the expression

$6(-3) = -18$

2) Find the value of $6a + 2a – 4a$ when $a = 2$

Step 1 Replace each letter with its assigned value

$6(2) + 2(2) – 4(2)$

Step 2 Perform the operation to find the value of the expression

$6(2) + 2(2) – 4(2)$

$= 12 + 4 – 8$

$= 8$

3) Evaluate the expression $2(x+4)$ when $x = 3$

Step 1 Replace each letter with its assigned value

$2(x+4)$

Step 2 Perform the operation to find the value of the expression

$2(x+4)$

$= 2(3 +4)$

$=2(7)$

$= 14$

NOW DO PRACTICE EXERCISE 2

Practice Exercise 2

1. If **x = 3** and **y = 4**, find values for the following expressions.

a) $\frac{x + 5}{y}$ b) $3y – 7$

c) $7x + 2y – 9$ d) $x + 3y + 8$

2. Simplify 6a + 4a and find its value when **a = 12.**

3. Write down in its simplest form:

a + a + a + b + b + b + b and find its value when **a = 5** and **b = 8**.

4. Write the following expressions in their simplest forms.

a) $15b + 11b$ b) $15x – 3x + 7x$

c) $9a – 4a + 6a + a$ d) $4x + 3x – 2x - x$

5. When **a = 2**, **b = 3**, find the numerical values of the following:

a) $3a + 2b + 1$ b) $5a – 3b + 6$

c) $6a + 2b – 3a + 1$ d) $4a – 5b – 2b + 12a$

6. Evaluate 4ab – 2ab + 6ab when **a = 4**, **b = 2**.

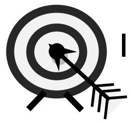
7. Find the numerical value of $a + \frac{1}{2}a + \frac{1}{4}a + \frac{1}{8}a$ when **a = 2**.

8. When **x = 4**, **y = 5**, **z = 1**, find the value of 3xy + 2yz – 3z – 1

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUBSTRAND 1

Lesson 3: Properties of Algebraic Expressions

In the previous lessons we translated mathematical phrases to algebraic expressions and evaluated them



In this lesson, you will:

- explain the meaning of each property associated with algebraic expressions.
- apply the property of algebraic expressions in solving a given problem

There are many laws which govern the order in which you perform operations in arithmetic and in algebra. We have looked at the order of operations in the last lesson. This lesson will be an extension to what we have learnt. We will study more on the algebraic properties. The three most widely discussed properties are:

1) Commutative Property of Addition and Multiplication
2) Associative Property of Addition and Multiplication
3) Distributive Property of Multiplication over Addition

COMMUTATIVE PROPERTY

Let us assume that a, b and c are numbers, variables or algebraic expressions.

- **Commute means to switch or change the order of numbers**
- **The Commutative Property of Addition states that a + b = b + a**
- **The Commutative Property of Multiplication states that a(b) = b(a)**
- **The Commutative Property does not work for subtraction and division.**

This law tells us that the order in which we add or multiply two numbers or algebraic expressions does not affect the sum or product. Switching or changing the order will not change the value of the expression.

Examples

The Commutative Property of Addition states that a + **b** = **b** + a

1) 3 + **4** = **4** + 3
2) **x^2** + x = x + **x^2**
3) 7 + 2 will give the same value as 2 + 7
4) 2x + 3x will give the same value as 3x + 2x
5) Simplify 3x – 4y + **2x** +7y = 3x + **2x** – 4y + 7y Commutative
   = 5x + 3y

The Commutative Property of Multiplication states that a(b) = b(a)

1) 3(4) = 4(3)
2) $x(x^2) = x^2(x)$
3) 7(2) will give the same value as 2(7)
4) 2x(3x) will give the same value as 3x(2x)

ASSOCIATIVE PROPERTY

- **Associate means to group together**
- **The Associative Property of Addition states that a + (b + c) = (a + b) + c**
- **The Associative Property of Multiplication states that a(b • c) = (a • b) c**
- **The Associative Property does not work for subtraction and division.**

Both addition and multiplication can actually be done with two numbers at a time. So if there are more numbers in the expression, how do we decide which two to "**associate**" first? The associative property of addition tells us that we can group numbers in a sum in any way we want and still get the same answer. The associative property of multiplication tells us that we can group numbers in a product in any way we want and still get the same answer.

Examples

The Associative Property of addition states that a + (b + c) = (a + b) + c

1) (2 + 3) + 6 = 2 + (3 + 6)
2) $x^3 + (2x + x) = (x^3 + 2x) + x$
3) (x + 2x)+ 3x = x + (2x + 3x)

The Associative Property of Multiplication states that a (b • c) = (a • b) c

1) 2(3y) = 3y(2)
2) 2(x • y) = 2x(y)

DISTRIBUTIVE PROPERTY

The distributive property of multiplication over Addition states that a(b + c) = a b + ac and (a + b) c = a c + b c

The distributive property comes into play when an expression involves both addition and multiplication. A longer name for it is, "the distributive property of multiplication over addition." It tells us that if a term is multiplied by terms in parenthesis, we need to "**distribute**" the multiplication over all the terms inside.

Even though order of operations says that you must add the terms inside the parentheses first, the distributive property allows you to simplify the expression by multiplying every term inside the parenthesis by the multiplier. This simplifies the expression.

Examples

1) a (b +c) = ab + ac

2) 2(x + 1) = 2x + 2

3) x(y – z) = xy – xz

4) 3x(2y – 7) = 6xy – 21x

ADDITIVE IDENTITY PROPERTY

This property tell us that any number or variable added to zero is that number or variable that is a + 0 = 0 + a = 0

Examples

1) b + 0 = b

2) 0 + 5 = 5

3) 2x + 0 = 2x

4) $3y^2 + 0 = 3y^2$

5) 3x + 2x + 0 = 3x + 2x = 5x

MULTIPLICATIVE IDENTITY PROPERTY

This property tells us that a number, variable or expression multiplied by 1 is equal to the number, variable or the expression.

That is a(1) = a.

Examples

1) 1 × 5 = 5

2) 2x(1) = 2x

3) 4y(1)(x) = 4yx

4) 5a(x + 1) = 5ax + 5a

We will now simplify algebraic expressions using a combination of all the properties discussed in this lesson.

Look at the examples on the next page.

Examples

1) $-6(a + 8) = -6a - 48$ Distributive Property

2) $6 + 3a + 2 + a = 6 + 2 + 3a + a$ Commutative Property
$= (6 + 2) + (3a + a)$ Associative Property
$= 8 + 4a$

3) $5a(x + 1) = 5ax + 5a(1)$ Distributive Property
$= 5ax + 5a$ Identity Property

4) $(1 - 7n)\,5 + 0 = 5 - 7n + 0$ Distributive Property
$= 5 - 7n$ Identity Property

5) Simplify $6 + t + 3$

$6 + t + 3 = (6 + t) + 3$
$= (t + 6) + 3$ Commutative Property
$= t + (6 + 3)$ Associative Property
$= t + 9$

6) Simplify $2a \times 3b$

$2a \times 3b = 2 \times a \times 3 \times b$ Expanded form
$= (2 \times a) \times (3 \times b)$ Associative Property
$= 2 \times 3 \times a \times b$ Commutative Property
$= (2 \times 3) \times (a \times b)$ Associative Property
$= 6 \times a \times b$
$= 6ab$

In practice all steps shown above may be performed at once.

7) Simplify $2a + 7 + 3$

$2a + 7 + 3 = 2a + (7 + 3)$
$= 2a + 10$

8) Simplify $(2a + 3) + 5$

$2a + (3 + 5) = 2a + 8$

9) Simplify $2 \times 3a$

$(2 \times 3) \times a = 6a$

NOW DO PRACTICE EXERCISE 3

Practice Exercise 3

1. Using the Commutative and Associative Properties. Simplify the numerical expressions. Write answers only.

   (a) $(3 + 4) + 97$

   (b) $(1 + 163) + 99$

   (c) $2 \times (9 \times 5)$

   (d) $4 \times (17 \times 25)$

2. Use the Commutative and Associative properties to write the algebraic expressions equivalent to the given ones. State in each case the property used.

   (a) $x + y$

   (b) xy

   (c) $t + 3$

   (d) $a + (b + c)$

   (e) $(x + y) + z$

   (f) $(2a + 3) + 5$

3. Simplify each expression

   (a) $x + 3 + 4$

   (b) $x + 3 + 0$

   (c) $x + 4 + 3 + 2$

   (d) $3 + 4 + t^2 + 5$

   (e) $2t + 5 + 3$

   (f) $x^2 + 2 + 7$

4. Use the Distributive Property to simplify each expression.

   (a) $4(1 + 9x)$

   (b) $8(-b - 4)$

   (c) $-6(x+ 4)$

   (d) $5(3m - 6)$

5. Simplify.

   (a) $2a + 7 + 3$

   (b) $x + 9 + 4$

   (c) $2x + 2 + 3x + 4$

   (d) $3 + 2b + 5b + 6$

   (e) $3ab + 9ab + 2ab$

   (f) $3x^2 + 2x^2 + 5x^2$

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1

Lesson 4: Simplifying Algebraic Expressions

In the previous lesson you learnt the laws and properties of algebraic expressions.

In this lesson, you will:

- define like and unlike terms
- identify like terms in a given algebraic expression
- simplify algebraic expression.

You were introduced to algebraic expressions in Grade 7. In the last lesson we looked at the order of operations and the properties of algebraic expressions. What you will learn in this lesson is helpful to simplify algebraic expressions.

What are algebraic expressions?

Algebraic expressions have numbers, variables and operational symbols.

Examples 1) $2n + 3$

2) $5x – 3y + 7$

3) $n^3 + n^2 + n$

> **Each part of an expression is separated from the rest of the expression by a plus (+) or minus (–) sign, and it is called a term**.

Examples

1) The expression $2n + 3$ has 2 terms, they are $2n$ and 3

2) The expression $5x – 3y + 7$ has 3 terms, they are $5x$, $3y$ and 7.

An algebraic expression such as $2n$ has two parts. The numerical part is the number and the literal part is the letter or variable.

Numerical coefficient → **2n** ← Literal coefficient

> Terms involving variables raised to the same exponent are called **like or similar terms.**

Example**:** $3x$, $2x$ and $5x$ are like or similar terms

x^3, $-7x^3$ and $9x^3$ are like terms

> Terms with different or with the same variables but raised to different exponents are called **unlike terms.**

Example: $7a$, $3b$ and $8c$ are unlike terms

$-6ab$, $4a^2b$ and $5a^2b^2$ are unlike terms

Examples

1) $6x$, x and $^{-}3x$ are called like terms because their literal parts are exactly the same.

2) $5a$, $2a$ and $4a$ are like terms

3) $2a^2$, a^2, $7a^2$ are like terms

4) $3xy$, $2xy$, yx are like terms

5) Terms such as $5x$ and $2y$ are called **unlike terms** because their literal parts are different.

6) $4a$ and $3b$ are unlike terms

7) $5x^2$ and $2y^2$ are unlike terms

8) ac and bc are unlike terms

When simplifying algebraic expressions, addition and subtraction can be applied on like terms. It is similar to adding or combining 2 boxes of powdered milk and 5 boxes of powdered milk. In symbols we write $2b + 5b = 7b$.

If we simplify expressions with a series of terms which are not all like, we can only indicate the addition and subtraction of these terms. It is also similar to adding or combining 3 xylophones, 2 yoyos and a zipper. In symbols we write $3x + 2y + z$.

Example 1 $a + 2a + 4x + 3a + 3x$

$= (a + 2a + 3a) + (4x + 3x)$ group the like terms

$= (1 + 2 + 3)a + (4+3)x$ add the numerical parts

$= 6a + 7x$ simplified answer

Example 2 $7b – 5b = (7 – 5)b$ add the numerical parts

$= 2b$

Example 3 $a + x + 2a + 3x + 2x$ group the like terms

$= (a + 2a) + (x + 3x + 2x)$

$= (1 + 2) a + (1 + 3 + 2) x$ add the numerical parts

$= 3a + 6x$ simplified answer

Example 4 Simplify $9b + b + 3b$

$9b + b + 3b = (9 + 1 + 3) b$

$= 13b$

Although expressions like x + y and 3b – 5c cannot be made any simpler, it is possible for an expression to contain several sets of like terms.

The expression can then be made simpler by adding or subtracting each set of like terms.

Example 5

Simplify $8p + 2q - 5p + 3q + 4p$

$8p + 2q - 5p + 3q + 4p$	collect and group like term
$= (8p - 5p + 4p) + (2q + 3q)$	
$= (8 - 5 + 4)p + (2 + 3)q$	add the numeral part
$= 7p + 5q$	final answer.

Example 6

Simplify $8t + 4s + 7s$

$8t + 4s + 7s = 8t + (4 + 7)s$	Group like terms
$= 8t + 11s$	add

Example 7

Simplify $7x - 5z + 3x + 8z$

$=(7x + 3x) + (8z - 5z)$	Group like terms
$= (7 + 3)x + (8 - 5)z$	add
$= 10x + 3z$	simplified answer

NOW DO PRACTICE EXERCISE 4

Practice Exercise 4

1 (a) Copy the expression $4x^2 + 6x + 2y + 8$

(b) How many terms are there?

(c) Circle the second term and put a box around the fourth term.

(d) Rewrite the expression with the terms in a different order.

---

2 Which of these pairs are like terms?

(a)	7e. 2e	(c)	16q, 17q	(e)	14g, 14	(g)	ab, 3ab
(b)	6, 6x	(d)	7y, 2y	(f)	$2x^2$, $4x^2$	(h)	2xy, 7xy

---

3 Add or subtract like terms.

(a)	$5x – 3x$	(c)	$12m – 7m$	(e)	$4x + 6x – 7x$
(b)	$7x – 2x$	(d)	$5xy + 6xy$	(f)	$3x^2 + 7x^2$

---

4 Simplify

Example: $6x – 3y + 4x$ $= 6x + 4x – 3y$
$= 10x – 3y$

(a)	$3a + 7a + 6b$	(c)	$2c – 4e + 5c$	(e)	$7a + 2a + b + 3b$
(b)	$7x – 5x + 6y$	(d)	$3p + 4q + 7p$	(f)	$7x + 3x + 8z – 5z$

5. In each of the following expressions, tell how many groups of like or similar terms are given then identify.

Example: $5x - 2y + x + y$
2 groups: ($5x$ and x); ($-2y$ and y)

(a) $8y + 3y + 4z - 6z$

(b) $6a + 2a + 10a$

(c) $9m^2 + 3n + 9n + 3m^2$

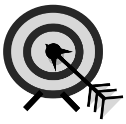
(d) $6mn + 3xy + 9 - 4mn + 7xy + 8$

(e) $8abc - 12 + abc - 12 + abc$

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUB STRAND 1

Lesson 5: Simplifying Algebraic Expressions involving Grouping Symbols

In the previous lesson you learnt how to simplify algebraic expressions by collecting like terms.



In this lesson, you will:

- simplify algebraic expression with grouping symbols.

We have discussed order of operations and the laws of simplifying algebraic expressions. This lesson will further help us with how to simplify algebraic expressions involving grouping symbols.

Whenever grouping symbols are involved, the symbols are first removed by applying the following rules.

RULE 1

To simplify an expression with grouping symbols preceded by a plus (+) sign, remove the grouping symbol and combine or collect like terms.

Examples

1) $2x + (3x + 4) = 2x + 3x + 4$
$= 5x + 4$

2) $2y + (4y – 3) = 2y + 4y – 3$
$= 6y – 3$

3) $x + (2x – 3y + 1) = x + 2x – 3y + 1$
$\mathbf{= 3x – 3y + 1}$

4) Simplify $2x + (3x – y) = 2x + (3x – y)$ remove brackets
$= 2x + 3x – y$ collect like terms
$\mathbf{= 5x – y}$

5) $9x^2 + (3x^2 – 5) = 9x^2 + 3x^2 – 5$
$= (9 + 3)x^2 – 5$
$\mathbf{= 12x^2 – 5}$

6) $(m + 3n – 5) + (-3m – 4n + 10) = m + 3n – 5 – 3m – 4n + 10$ remove brackets
$= m – 3m + 3n – 4n – 5 + 10$ collect like terms
$= (1 – 3)m + (3 – 4)n + (-5 + 10)$
$\mathbf{= -2m – n + 5}$

RULE 2

To simplify an expression with a grouping symbol preceded by a minus (–) sign, remove the grouping symbol and change the signs of the terms inside the grouping symbol to their opposite (positive to negative or negative to positive), then collect or combine like terms.

Example 1 $2x - (3x + 4) = 2x - 3x - 4$
$= (2 - 3)x - 4$
$\mathbf{= -x - 4}$

Example 2 $-2x^2 - (3x^2 - 4) = -2x^2 - 3x^2 + 4$
$= (-2 - 3)x^2 + 4$
$\mathbf{= -5x^2 + 4}$

Example 3 $9x^2 - (3x^2 - 5) = 9x^2 - 3x^2 + 5$
$= (9 - 3)x^2 + 5$
$\mathbf{= 6x^2 + 5}$

Example 4 $2x - (5y + 6) - 5 = 2x - 5y - 6 - 5$

$\mathbf{= 2x - 5y - 11}$

Example 5 Simplify the expression $-(m + 3n - 5) - (-3m - 4n + 10)$.

Solution: $-(m + 3n - 5) - (-3m - 4n + 10)$
$= -m - 3n + 5 + 3m + 4n - 10$ remove grouping symbols
$= -m + 3m - 3n + 4n + 5 - 10$ collect like terms
$= (-1 + 3)m + (-3 + 4)n + (5 - 10)$
$\mathbf{= 2m + n - 5}$

Now let us apply the two rules in the following expressions:

Example 6 $8a + (3a + 2) - (4a + 3)$
$= 8a + 3a + 2 - 4a - 3$ remove grouping symbols
$= 8a + 3a - 4a + 2 - 3$ collect like terms
$\mathbf{= 7a - 1}$

Example 7 $-(9y^2 + 3y - 4) + (4y + 7) - (7y^2 + 5 - 2y)$
$= -9y^2 - 3y + 4 + 4y + 7 - 7y^2 - 5 + 2y$ remove grouping symbols
$= -9y^2 - 7y^2 - 3y + 4y + 2y + 4 + 7 - 5$ collect like terms
$\mathbf{= -16y^2 + 3y + 6}$

RULE 3

To simplify an expression involving a series of grouping symbols, start by removing from the innermost grouping symbol and apply Rules 1 and 2.

Example 1	$3x + [3 + (4x + 2)]$	$= 3x + [3 + 4x + 2]$	remove parentheses
		$= 3x + 3 + 4x + 2$	remove brackets
		$= 3x + 4x + 5$	collect like terms
		$\mathbf{= 7x + 5}$	**Answer**

Example 2	$2x – \{3y + [-8 – 5y – (x – 4)]\}$	
	$= 2x – \{3y + [-8 – 5y – x + 4]\}$	remove parentheses
	$= 2x – \{3y – 8 – 5y – x + 4\}$	remove brackets
	$= 2x – 3y + 8 + 5y + x – 4$	remove braces
	$= 2x + x – 3y + 5y + 8 – 4$	collect like terms
	$\mathbf{= 3x + 2y + 4}$	**Answer**

Example 2	$4y + y\,[5y + (x + 2) + 10]$	remove parentheses
	$= 4y + y\,[5y + x + 2 + 10]$	collect like terms
	$= 4y + y\,[5y + x + 12]$	bracket removed
	$\mathbf{= 4y + 5y^2 + xy + 12y}$	**Answer**

Example 3	Simplify	$3(4a – b) – 2(3a – 2b)$	
		$= 3(4a – b) – 2(3a – 2b)$	remove grouping symbols
		$= 12a – 3b – 6a + 4b$	collect like terms
		$\mathbf{= 6a + b}$	**Answer**

NOW DO PRACTICE EXERCISE 5

Practice Exercise 5

1. Remove the grouping symbol.

a) $6(a + 2)$ (b) $-2(p – q)$ (c) $-(c + 2)$ (d) $(p + 4)3$

2. Remove grouping symbols and simplify where possible by collecting like terms.

Example: $2(x + 4) + 3x = 2x + 8 + 3x$
$= 5x + 8$

a) $2(x + 5) + 8$ (b) $3a – 2(a + b)$ (c) $-6(x + y) + 2y$

3. Remove the grouping symbols and simplify where possible by collecting like terms.

Example: $3(2y – 4) + 2(y + 5) = 6y – 12\ 2y + 10$
$= 8y – 2$

a) $2(a + b) + (3a – b)$ b) $5(2x^2 + 3) + 2x\,(3 – 2x)$

4. Remove the parentheses and simplify if possible.

a) $4(p + 2q) + 5(3p – 4q)$ b) $3(a – 2b) + 5(2a + 3b)$

c) $3(x + 3y) + 8x$ d) $15p + 3(p – 8q)$

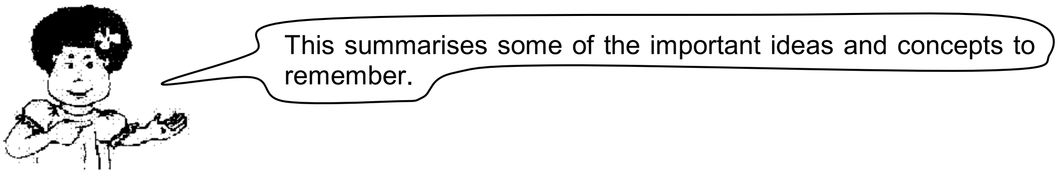
5. Simplify

a) $x – y + [\ -8 – 5y – (x – 4)\ ]$

b) $\{3y + 8 – [5y – (x + 2)]\} + 2x$

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1

SUB–STRAND 1: SUMMARY



- A variable is a letter which takes the place of a number.
- A constant is a number with a fixed value.
- A numerical expression may consist of a single number with or without operational symbols, or two or more numbers with operation and grouping symbols.
- An algebraic expression may consist of numbers and one or more variables joined by a symbol of operation and sometimes uses grouping symbols.
- To evaluate an algebraic expression, substitute or replace the letter with a numerical value.
- The commutative law of addition states that $y + z = z + y$, this law says that the order in which you add a number does not matter.
- The commutative law of multiplication states that the order in which you multiply a number does not matter.
- The commutative law does not apply to subtraction and division.
- The Associative law of addition states that no matter how terms are grouped in carrying out additions, the sum will always be the same. $(a + b) + c = a + (b + c)$
- The Associative law of multiplication states that no matter how terms are grouped in carrying out multiplications, the product will always be the same. $(a \times b) \times c = a \times (b \times c)$
- The associative law does not apply to subtraction and division.
- The distributive law states the difference between addition and multiplication that is $x(y + z) = xy + xz$
- The zero property states that the result of zero added to a number is that number. e.g. $3 + 0 = 0$
- The multiplicative identity property states that multiplying 1 by a number results in that number. e.g. $20 \times 1 = 20$
- Only like terms can be added or subtracted.
- Simplifying algebraic expressions involve removing and adding or collecting of like terms.

REVISE LESSONS 1- 5 THEN DO SUB-STRAND TEST 1IN ASSIGNMENT 6.

ANSWERS TO PRACTICE EXERCISES 1 - 5

Practice Exercise 1

1. (a) 4 (b) 7 (c) 6

2. 6 is the constant

3. (a) x + 3
   (b) z – 5
   (c) 2a
   (d) x + 4
   (e) z ÷ 4 or $\frac{z}{4}$
   (f) abc

4. (a) iv
   (b) v
   (c) ii
   (d) iii
   (e) i

5. increase a number by three or three more than a number.

6. (a) numerical
   (b) algebraic

Practice Exercise 2

1. (a) 2 (b) 5
   (c) 28 (d) 23

2. 10a = 120

3. 3a + 4b = 47

4. (a) 26b (b) 19x
   (c) 12a (d) 4x

5. (a) 13 (b) 7
   (c) 13 (d) 11

6. $8ab = 64$

7. $\frac{15}{8}a = \frac{15}{4}$ or 3.75

8 66

Practice Exercise 3

1. (a) 104 (b) 263 (c) 90 (d) 1700

2. (a) $y + x$ Commutative Property
   (b) xy Commutative Property
   (c) $3 + t$ Commutative Property
   (d) $(a + b) + c$ Associative Property
   (e) $x + (y + z)$ Associative Property
   (f) $2a + (3 + 5)$ Associative Property

3. (a) $x + 7$ (b) $x + 3$ (c) $x + 9$ (d) $12 + t^2$
   (e) $2t + 8$ (f) $x^2 + 9$

4. (a) $4 + 36x$ (b) $-8b – 32$ (c) $-6x – 24$ (d) $15m – 30$

5. (a) $2a + 10$ (b) $x + 13$ (c) $5x + 6$ (d) $7b + 9$
   (e) $14ab$ (f) $10x^2$

Practice Exercise 4

1. (a) $4x^2 + 6x + 2y – 8$
   (b) 4 terms
   (c) $4x^2 + 6x + 2y – \boxed{8}$
   d) $4x^2 + 2y + 6x – 8$

2. (a) like (b) unlike (c) like (d) like
   (e) unlike (f) like (g) like (h) like

3. (a) $2x$ (b) $5x$ (c) $5m$ (d) $11xy$
   (e) $3x$ (f) $10x^2$

4. (a) $10a + 6b$ (b) $2x + 6y$ (c) $7c – 4e$ (d) $10p + 4q$
   (e) $9a + 4b$ (f) $10x + 3z$

5.	(a)	2 groups:	(8y and 3y); (4z and – 6z)
	(b)	1 group:	(6a, 2a and 10a)
	(c)	2 groups:	($9m^2$ and $3m^2$); (3n + 9n)
	(d)	3 groups:	(6mn and – 4mn); (3xy and 7xy); (9 and 8)
	(e)	2 groups:	(8abc, abc and abc); (–12 and –12)

Practice Exercise 5

1.	(a)	6a + 12	(b)	-2p + 2q	(c)	-c – 2	(d)	3p + 12
2.	(a)	2x + 18	(b)	a – 2b	(c)	-6x – 4y		
3.	(a)	5a + b	(b)	$10x^2 + 2x + 15$				
4	(a)	19p – 12q	(b)	13a + 9b	(c)	11x + 9y	(d)	18p – 24q
5	(a)	-6y – 4	(b)	-2y + x + 12				

END OF SUB STRAND 1

SUB-STRAND 2

INDICES

SUB STRAND 2: INDICES

Introduction

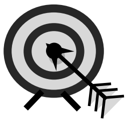
This Sub strand involves indices. Indices are basically a repeated multiplication of expressions. It has rules or laws that are required to be followed to simplify expressions. If the laws are studied well you can be able you solve difficult questions.

In this sub strand, you will:

- define the terms exponent, power and base
- simplify exponents by using rules appropriately and accurately
- write expressions in index notation and vice versa
- perform indicated operations correctly to get the right answer.
- change negative exponents to positive exponents

Lesson 6: Exponents and Powers

In the previous lessons you learnt about algebraic expressions and how you can simplify algebraic expressions.



In this lesson, you will:

- define exponents, base, power and coefficients
- determine the base, exponent, power and coefficient
- convert expressions from expanded form into index notation form.

This lesson is an extension to what you learnt in grade 7. You learnt that indices or powers are an important part of mathematics as they are necessary to indicate that a number is multiplied by itself for a given number of times.

In Grade 7, we defined exponent, base and power as,

- **Exponent is another way of writing the multiplication of the same factor repeatedly. It is a number or variable written on the upper right of the variable or expression that indicates the number of times the number or expression is to be multiplied by itself.**
- **The base indicates the factor that is to be multiplied.**
- **The power is the product of the equal factors.**

Examples

1) The multiplication statement 2 x 2 x 2 x 2 x 2 = 2^5 shows 2 as the base, 5 is the exponent while 2^5 is the power. It is usually read as two to the fifth power. It can also be evaluated as the fifth power of 2 is 32.
2) The numerical expression 6^2 means 6 x 6. The exponential form 6^2 is read as 6 to the second power or the second power of 6 is 36
3) The exponential x^4 is read as x to the fourth power.
4) $(\frac{1}{3})^3$ is read as $\frac{1}{3}$ to the third power which is $\frac{1}{27}$

Here is a table showing more examples

Expanded expression	**Index notation**	**Exponent**	**Base**	**Power**
2 x 2 x 2	2^3	3	2	Third power of 2 is 8
8 x 8 x 8 x 8	8^4	4	8	The fourth power of 8 is 4096
a x a x a x a x a	a^5	5	a	The fifth power of a is a^5

The third example in the table on page 41 can be further evaluated if the numerical value of **a** is known.

You will have noticed that in the second column the term index notation is used instead of exponential form as you have learnt in Grade 7.

Index notation or **exponential form** is a short way of writing multiplication of the same factors repeatedly the given number of times.

That is,

$$\mathbf{2 \times 2 \times 2 \times 2 \times 2 = 2^5}$$

Expanded form Index notation

The index notation consists of a base and an exponent. The following examples show how an expanded expression can be written in short using index notation.

1) $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$
2) $a \times a \times a \times a \times a \times a \times a \times a = a^8$
3) $y \times y \times y \times y \times 4 \times 4 = y^4 \times 4^2 =$
4) $5 \times m \times n \times n \times n = 5mn^3$
5) $5 \times 5 \times 7 \times 7 \times 7 = 5^2 \times 7^3$

Expressions written in index notation can be written in expanded form too. The exponent tells you how many times the base is to be multiplied by itself.

Examples

1) $y^5 = y \times y \times y \times y \times y$
2) $b^3 = b \times b \times b$
3) $7p^2q^3 = 7 \times p \times p \times q \times q \times q$
4) $8mn^4 = 8 \times m \times n \times n \times n \times n$
5) $a^2 + c^2 = a \times a + c \times c$
6) $\dfrac{c^5}{ab^3} = \dfrac{c \times c \times c \times c \times c}{a \times b \times b \times b}$

What is Coefficient?

Coefficient is the numerical part in front of a term or an algebraic expression.

Examples

1) In $5x - 2y$, 5 is the coefficient of x and -2 is the coefficient of y.

2) In $2z$, 2 is the coefficient of z.

3) In $5x^2$, 5 is the coefficient of x^2.

4) $10s^3$, 10 is the coefficient of s^3.

Here is a table showing more examples.

Expression	Coefficients
$3x^2 + x - 2$	Coefficient of x^2 is 3 and x is 1
$\frac{1}{3}a + 2$	Coefficient of a is $\frac{1}{3}$
$2x + 5y$	Coefficient of x is 2 and of y is 5

NOW DO PRACTICE EXERCISE 6

Practice Exercise 6

1. Write each of these in a shorter form by using index notation.

   a) $2 \times 2 \times 2 \times 3 \times 3$
   b) $6 \times b \times b$
   c) $4 \times y \times y \times y$
   d) $a \times a \times b \times b \times b \times b$

2. Identify the coefficients in the expression $3x^2 - 2x + 1$.

3. What is the

   a) third power of 2?
   b) fourth power of 3?
   c) first power of 8?

4. What does each symbol mean?

   a) x^5
   b) 5^3
   c) $(5a)^3$
   d) $5a^3$

5 Given $3^4 = 81$, which number is called the

   a) base?
   b) power?
   c) exponent?

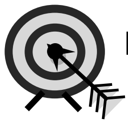
6. Write the meaning of these symbols.

   a) a^2a^3
   b) $(ab)^3$
   c) $(a^2)^3$

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUB STRAND 2

Lesson 7: The Product Law

In the previous lesson you learnt how to write expressions in index or exponential notation.



In this lesson, you will:

- state the product Law and rule formatting???
- use the product rule when multiplying expressions with exponents of the same base.

In the previous lessons, you learnt that a long expression can be written in short using the index notation.

Example $2 \times 2 \times 2 = 2^3$

You also learnt about the base and the exponent, that is

base ⟶ $\mathbf{2^3}$ ⟵ exponent

Expressions with exponents can be multiplied by following the Product law.

> The Product law states that when multiplying exponents with the same base, we add the indices.

$$\boxed{\mathbf{a^n \cdot a^m = a^{n+m}}}$$

We can solve easily the product of 2 powers of **a** by adding the two exponents of a

Example 1 $a^3 \cdot a^2 = (a \cdot a \cdot a) \cdot (a \cdot a) = a \cdot a \cdot a \cdot a \cdot a$

factor **a** is used 3 times | factor **a** is used 2 times | factor **a** is used 5 times

Hence, $a^3 \cdot a^2 = a^{3+2} = a^5$

Example 2 $a^2 \cdot a \cdot b \cdot b^2 \cdot b^3$

$= (a^2 \cdot a) \cdot (b \cdot b^2 \cdot b^3)$ group exponents with the same bases

$= a^{2+1} \times b^{1+2+3}$ add the exponents

$= a^3b^6$

Example 3 $ab^3 \cdot a^2b$

$= a^{1+2} \cdot b^{3+1}$ add the indices

$= a^3b^4$

Example 4 $a^m \bullet a^m \bullet a^n \bullet a^n$

$= a^{m+m+n+n}$ add the indices

$= a^{2m+2n}$

Apart from the product law you must take note of how you can add or subtract like and unlike terms which you learnt in sub-strand 1 as well as other algebraic properties.

The following examples show how algebraic expressions or exponents that contain coefficients can be simplified.

1) $(2y^3)(3y)$

$= 2 \bullet 3 \bullet y^{3+1}$ multiply the coefficients and add the indices of the same base

$= 6y^4$

2) $(2a^4)(2a^5)$

$= 2 \bullet 2 \bullet a^{4+5}$ multiply the coefficients and add indices

$= 4a^9$

3) $(2a^7)(a^3)$

$= 2a^{7+3}$

$= 2a^{10}$

4) $(5b^2)(3b^5)$

$= 5 \bullet 3 \bullet b^2 \bullet b^5$

$= 15 \bullet b^{2+5}$

$= 15b^7$

NOW DO PRACTICE EXERCISE 7

Practice Exercise 7

1. Simplify.

a) $(a^2)\ (a^3)$ c) $(m^2)\ (m^6)$ e) $(y^2)\ (y^3)\ (y)$

b) $(b^4)\ (b^3)$ d) $(y^3)\ (y)$ f) $(a^5)\ (a^3)$

2. Simplify.

a) $(5a)\ (3a^2)$ c) $(5x^2)\ (6x^2)$ e) $(2a^5)\ (3a^2)$

b) $(5y^3)\ (6y^4)$ d) $(2b)\ (6b^3)$ f) $4b^3)\ (3b^3)$

3. Simplify.

a) $(2ab)\ (ab)$ d) $2b)\ (3b^3)$

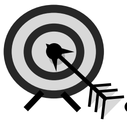
b) $(x^2y)\ (xy^2)$ e) $(7x^3a)\ (x^3a)$

c) $(2a)\ (3a^2)\ (a^3)$

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUBSTRAND 2

Lesson 8: The Power Law

In the previous lesson you learnt how to use the product rule. This rule will be needed in this lesson as well.



In this lesson, you will:

- state the power law
- use the power law when raising a power to another exponent.

The product law states that we can multiply exponents with the same base by adding the indices.

Example 1) $a \times a^2 = a^3$ by the product law

An exponent such as $(a^4)^3$ can be solved the same way using the Product law. $(a^4)^3$ means the power inside the parentheses is multiplied by itself three times. We can expand the expression and use the product law to simplify it.

Here is how you can simplify

$$(a^4)^3 = a^4 \bullet a^4 \bullet a^4$$

$$= a^{4+4+4} \quad \text{by the Product law}$$

$$= a^{12}$$

Such expressions are made even simpler by the Power Law.
The power law states that we multiply the indices when a power is raised to an exponent.

$$\boxed{\mathbf{(x^a)^b = x^{ab}}}$$

Examples

1) $(a^4)^3 = a^{4 \times 3}$ multiply the powers

$= a^{12}$

2) $(7^2)^2 = 7^{2 \times 2}$

$= 7^4$

3) $(y^3)^2 = y^{3 \times 2}$

$= y^6$

Numerical expressions like example 2 on page 48 can be simplified further by finding its value. This is how it can be done.

Solution $(7^2)^2 = 7^{2 \times 2}$

$= 7^4$ $(7^4 = 7 \times 7 \times 7 \times 7)$

$= 2401$

Now see some more examples.

4) $(3^2)^2 = 3^{2 \times 2}$ by Power law

$= 3^4$

$= 81$ $(3^4 = 3 \times 3 \times 3 \times 3)$

5) Calculate the value $(2^2)^3 - (2^2 \bullet 3)$

$(2^2)^3 - (2^2 \bullet 3)$

$= 2^{2 \times 3} - (2 \bullet 2 \bullet 3)$

$= 2^6 - 12$

$= 64 - 12$

$= 52$

6) $(a^2b^2)^3$ means the same as $(a^2b^2)\ (a^2b^2)^3\ (a^2b^2)^3$

$= a^{2 \times 3}\ b^{2 \times 3}$ by the power law

$= a^6b^6$

7) $(3a^3)^2 = 3a^3 \bullet 3a^3$

$= 3^2a^{3+3}$

$= 3^2a^6$

$= 9a^6$

The next example requires a slightly longer step.

8) $a^4c^4 \bullet a^6c^7 = a^4 \bullet c^4 \bullet a^6 \bullet c^7$

$= a^4 \bullet a^6 \bullet c^4 \bullet c^7$ collect all the like terms

$= a^{4+6} \bullet c^{4+7}$ then add the indices

$= a^{10} \bullet c^{11}$ simplify

$= a^{10}\ c^{11}$

NOW DO PRACTICE EXERCISE 8

Practice Exercise 8

1. Using the Product law to simplify.

a) $b^4 \times b^7 \times b^3$ b) $p^5 \times p^8 \times p^4 \times p \times p^7$

2. Write these statements as simply as possible.

a) $a^4 \times a^3$ b) $b^7 \times b^4$ c) $c^5 \times c^3$

3. Write these products as a single power.

a) $2^4 \times 2^5 \times 2^3$ b) $3^2 \times 3 \times 3^4$

4. Simplify the following

a) $(m^2)^3$ b) $(x^2)^5$ c) $(y^6)^2$

5. Copy and complete each of the following.

a) $(mn)^4 = m^{\square}n^4$ b) $(cd)^7 = c^7d^{\square}$

c) $(x^5y^2)^3 = x^{\square}y^{\square}$ d) $(g^2h)^5 = g^{\square}h^{\square}$

e) $(2p)^9 = 2^{\square}p^{\square}$ f) $(3j^5)^2 = 3^{\square}j^{\square}$

g) $(5w)^6 = 5^{\square}w^{\square}$ h) $(8f^3)^3 = 8^{\square}f^{\square}$

6. Simplify

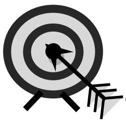
a) $(2^2)^3 + (2^2 \text{ x } 3)$ b) $2(m^2)^3$

c) $(2a^2)^2$

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUB STRAND 2

Lesson 9: The Quotient Law

In the previous lesson, you learnt the product rule. You are required to master the product rule so that you can apply it to all the situations that deal with exponents.



In this lesson, you will:

- state the quotient law
- use the quotient rule in exponents when dividing the same base.

You have learnt about the exponent, the product and the power law. In this lesson you will be introduced to the quotient law and you will learn to use it to solve algebraic expressions involving exponents. It is important that you master the previous laws together with the quotient law.

The product and the power laws state that:

1) $\mathbf{x^a \bullet x^b = x^{a+b}}$

2) $\mathbf{(x^a)^b = x^{a \times b}}$

Before we look at the quotient law, let us solve the following problem by expansion and simplification.

Find the value of $3^4 \div 3^2$

Solution:

$3^4 \div 3^2 = \dfrac{\cancel{3} \times \cancel{3} \times 3 \times 3}{\cancel{3} \times \cancel{3}}$ expand numerator and denominator

$= 3 \times 3$ simplify (3 x 3 left in the numerator)

$= 9$

After simplifying the factors of the denominator with factors in the numerator, there are factors 3 x 3 left in the numerator.

Such a problem can be done simply by using the **QUOTIENT RULE.**

The rule states that when dividing index expressions with the same base SUBTRACT the powers.

$$\mathbf{\frac{x^a}{x^b} = x^{a-b}}$$

Examples

1) $a^7 \div a^5 = a^{7-5} = a^2$

The same question can be written in another way and solved using the same rule

$$\frac{a^7}{a^5} = a^{7-5} = a^2$$

2) $x^7 \div x^3 = x^{7-3}$

$= x^4$

5) $\frac{12y^3}{4y} = \frac{12}{4} \bullet \frac{y^3}{y}$ Simplify the numeral and use quotient law

$= 3 \bullet y^{3-1}$
$= 3y^2$

6) $\frac{pq}{p} = p^{1-1}q$ Quotient law

$= p^0q$

$= 1 \bullet q$

$= q$

7) $4a^7 \div a^2 = \frac{4a^7}{a^2} = 4a^{7-2} = 4a^5$

8) $\frac{3m^5n^2}{9m^3} = \frac{3}{9}m^{5-3}n = \frac{m^2n^2}{3}$

9) $\frac{12x^9y^6}{8xy^3} = \frac{12}{8}x^{9-1}y^{6-3}$

$= \frac{3}{2}x^8y^3$

NOW DO PRACTICE EXERCISE 9

Practice Exercise 9

1. Simplify and give your answer in index form

a) $3^6 \div 3^2$ b) $5^4 \div 5^3$ c) $10^6 \div 10^4$

d) $8^3 \div 8^2$ e) $4^{12} \div 4^6$ f) $2^8 \div 2$

g) $5^7 \div 5^3$ h) $10^7 \div 10$ i) $a^4 \div a^3$

j) $y^8 \div y^2$ k) $m^5 \div m$ l) $x^9 \div x^3$

m) $t^2 \div t^6$ n) $a^7 \div a^{14}$ o) $x^6 \div x^{11}$

---

2. Simplify

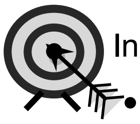
a) $\dfrac{2r^4}{r^2}$ b) $\dfrac{6p^6}{12p^3}$ c) $\dfrac{15r^8}{5r^6}$

d) $\dfrac{a^7b^6c^3}{a^6b^4c}$ e) $\dfrac{(2r^4)(3r^5)}{r^2}$ f) $\dfrac{(6p^6)(4p^4)}{12p^3}$

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUBSTRAND 2

Lesson 10: Raising a Quotient to a Power

In the previous lesson you learnt about „The Quotient Law‟ and used quotient law in exponents when dividing the same bases.



In this lesson, you will:

- use a combination of the product, quotient and the power rule in exponents when raising a quotient to a power.

You have learnt the Product, Power and the Quotient law in the previous lessons. You are required to learn the previous laws well to use them in this lesson. The laws learnt were:

a) $\mathbf{x^a \bullet x^b = x^{a+b}}$
b) $\mathbf{(x^a)^b = x^{(a)(b)}}$
c) $\mathbf{x^a \div x^b = x^{a-b}}$

To simplify a quotient raised to a power, expand the expression then simplify by using the product, power or the quotient laws. A quotient raised to a power indicates that both the numerator and the denominator are raised to that same power.

$$\left(\frac{x^a y^b}{z^c}\right)^n = \frac{x^{an} y^{bn}}{z^{cn}}$$

Examples

1) $\left(\frac{2a^2}{a}\right)^4 = \frac{(2^4)(a^2)^4}{a^4}$ Power law

$= \frac{2^4 a^8}{a^4}$ Quotient law

$= 16a^{8-4}$

$= 16a^4$

Notice the whole expression (the numerator and the denominator) in the parentheses is raised to the power of 4 and the numerical exponents are evaluated.

2) Simplify $(2a^2 \div a)^4$

Solution: $(2a^2 \div a)^4$

$= 2^4a^{2 \times 4} \div a^4$ inner expression is raised to 4

$=16a^8 \div a^4$ Quotient law

$= 16a^{8-4}$

$= 16a^4$

Example 1 and 2 shows the same question solved in two different ways.

3) $\left(\dfrac{4x^3y^2}{4xy}\right)^6 = \dfrac{4^6x^{3x6}\ y^{2 \times 6}}{4^6x^6y^6}$ Power law

$= \dfrac{4^6x^{18}y^{12}}{4^6x^6y^6}$ Quotient law

$= x^{18-6}\ y^{12-6}$

$=x^{12}y^6$

4) $\left(\dfrac{b^2}{b}\right)^3 = \dfrac{b^{2 \times 3}}{b^3}$ Power rule

$= \dfrac{b^6}{b^3}$ Quotient law

$= b^{\ 6-3}$

$= b^3$

5) Simplify $\left(\dfrac{x^4}{y^2}\right)^2$

$= \dfrac{x^{4 \times 2}}{y^{2x2}}$ Power law

$= \dfrac{x^8}{y^4}$

The next lot of examples are slightly different to the above examples. The expressions require the combination of laws to simplify.

6) Simplify $\dfrac{a^3 \text{ x } a^5 \text{ x } a^6}{a^2 \text{ x } a^4} = \dfrac{a^{3+5+6}}{a^{2+4}}$ Product law

$= \dfrac{a^{14}}{a^6}$ Quotient law

$= a^{14-6}$

$= a^8$

7) Simplify $x^6 \div x^4$.

$x^6 \div x^4 = x^{6-4} = x^2$ Quotient law

8) Simplify $x^6 y^{10} \div x^4 y^2$.

$x^6 y^{10} \div x^4 y^2 = \dfrac{x^6 y^{10}}{x^4 y^2}$ Quotient law

$= x^{6-4} \times y^{10-2}$

$= x^2 y^8$

9) Simplify $8x^{10} \div 2x^4$.

$= \dfrac{8x^{10}}{2x^4}$

$= \dfrac{8}{2} x^{10-4}$ (note: 8÷ 2, divide the numerals = 4)

$= 4x^6$

10) Simplify $3x^{12} \div 18x^4$.

$= \dfrac{3x^{12}}{18x^4}$

$= \dfrac{1}{6} x^{12-4}$

$= \dfrac{1}{6} x^8$

Examples 8 and 9 involved coefficients, notice that the coefficients are numerals and they simplify each other.

NOW DO PRACTICE EXERCISE 10

Practice Exercise 10

1. Simplify and give answers in simplest index form.

(a) $a^6 \times a^7 \times a^3$ (b) a^4 x a^3 x a^2

(c) $2a^4b^7$ x $3a^2b$ x $3ab^4$ (d) $3s^6y^7$ x $3sy$ x 3^2s^2y

2. Simplify

(a) $\dfrac{15x^6}{25x^3}$ (b) $\dfrac{16a^7}{42a^5}$

(c) $\dfrac{2^6 a^4 b^{13} c^8}{2^4 a^3 b^7 c^5}$ (d) $\dfrac{3^4 a^7 b^5 c^{10}}{3^4 ab^4 c^5}$

3. Simplify

(a) $(a^4)^3$ (b) $(m^3)^7$

(c) $(2x^4)^2$ (d) $(9x^3)^2$

(e) $(4x^2y^3)^4$ (f) $(3m^2n^2p^3)^3$

(g) $\left(\dfrac{x}{y}\right)^2$ (h) $\left(\dfrac{y}{z}\right)^2$

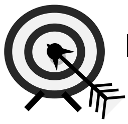
(i) $\left(\dfrac{3x^4}{y^2}\right)^3$ (j) $\left(\dfrac{4m^2}{n^2}\right)^5$

(k) $\left(\dfrac{9a^2b^3}{c^3}\right)^4$ (l) $\left(\dfrac{7m^2n^3}{p^5}\right)^2$

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 2

Lesson 11: Zero Indices

In the previous lesson you learnt about "Raising A Quotient to a Power" using a combination of the product rule, power rule and quotient rule in exponent when raising a quotient to a power.



In this lesson, you will:

- use the zero rule to simplify given expressions.

You have studied the laws of indices throughout this sub-strand. These laws together with the zero indices which you will study in this lesson are important laws that you must be very familiar with because you need them to simplify algebraic expressions to their simplest forms.

To investigate the meaning of Zero index, we will simplify $3^2 \div 3^2$ in two different ways

The first way is to expand and simplify.

$$3^2 \div 3^2 = \frac{3 \times 3}{3 \times 3} = \ =1$$

Next we will use the Quotient law.

$$3^2 \div 3^2 = 3^{2-2}$$

$$= 3^0$$

It is the same problem solved in two different ways. In the first method the answer is 1, in the second method the answer is 3^0. We can therefore conclude that $3^0 = 1$.

Generally any expression, variable or term raised to the power of 0 is equal to one.

$$\mathbf{x^0 = 1} \qquad\qquad \mathbf{(ax^m)^0 = 1}$$

Examples

1) Thus $2^0 = 1$

2) $5^0 = 1$

3) $(2a)^0 = 1$

4) $2a^0 = 2 \times 1 = 2$

In example 4, **a** raised to 0 gives an answer of 1 multiplied by 2 gives 2.

5) Simplify $a^6 \div a^6$

Solution: $a^6 \div a^6 = a^{6-6}$ by quotient law

$= a^0$ zero index

$= 1$

6) Simplify $3(x^2y^7z)^0$

Solution: $3(x^2y^7z)^0 = 3 \times 1$ zero index

$= 3$

Expressions can be long and may look difficult to simplify. It is therefore wise to simplify following the order of operations, the algebraic and index laws. The zero index law is simple and can be worked out quickly by inspection.

The next lot of examples are long and need careful consideration when simplifying.

7) $6^3k^4 \div 6^2k^4$

$= 6^{3-2}\, k^{4-4}$ Quotient law

$= 6k^0$ zero index

$= 6$

8) $5(5^2x^4y)^0 = 5 \times 1$ zero index

$= 5$

9) $7^2(a^3b^4)^0 - 6^2$

$= 49 \times 1 - 36$ zero index

$= 49 - 36$

$= 13$

10) $(2x + y)^0 + 2^0$

$= 1 + 1$ zero index

$=2$

NOW DO PRACTICE EXERCISE 11

Practice Exercise 11

1. Simplify each of the following.

a)	K^0	b)	W^0	c)	10^0
d)	6^0	e)	$7x^0$	f)	$3y^0$
g)	$7 + 9^0$	h)	$3 + y^0$	i)	$a^0 b^2$
j)	$m^0 n^4$	k)	$c^3 d^0 e^0$	l)	$x^7 y^0 2^0$
m)	$\frac{km^0}{n^0}$	n)	$\frac{ab^0}{c^0}$	o)	$-3 a^4 b^0$
p)	$-8g^{11}h^0$				

2. Which is larger?

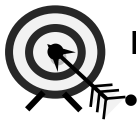
a) 10^0 or 100^0.

b) $6k^0$ or $7k^0$.

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 2

Lesson 12: Negative Indices

In the previous lesson you have learnt about the "Zero Indices". You also simplified expressions with the zero indices by using zero index rules.



In this lesson, you will:

- change negative indices into positive indices by moving a factor from the numerator to the denominator or from the denominator to the numerator.
- simplify expressions with negative indices.

Let us investigate what we mean by Negative Indices.

The table below shows how $\frac{3^2}{3^4}$ can be simplified in two different ways.

By Quotient law	By expanding
$\frac{3^2}{3^4} = 3^{2-4} = 3^{-2}$	$\frac{3^2}{3^4} = \frac{3\times3}{3\times3\times3\times3} = \frac{1}{3\times3} = \frac{1}{3^2}$

From the two methods we can conclude that $3^{-2} = \frac{1}{3^2}$.

3^{-2} has a negative exponent and therefore is called a **negative index** while $\frac{1}{3^2}$ has a positive exponent and is called a positive index.

In general

$$\boldsymbol{a^{-m} = \frac{1}{a^m} \quad \text{or} \quad a^m = \frac{1}{a^{-m}} \quad \text{where } a \neq 0}$$

Examples

1) $3^{-1} = \frac{1}{3}$ 2) $5^{-2} = \frac{1}{5^2}$ 3) $4^6 = \frac{1}{4^{-6}}$

The next examples show how you can use the zero index law to change exponents from positive to negative or vice versa.

4) Write $\frac{1}{5^3}$ as a negative index.

Solution $\frac{1}{5^3} = \frac{5^0}{5^3}$ by the zero index law $5^0 = 1$

$= 5^{0-3}$ quotient law

$= 5^{-3}$

5) Write $\frac{1}{5^{-3}}$ as a positive index.

Solution $\frac{1}{5^{-3}} = \frac{5^0}{5^{-3}}$ zero index law

$= 5^{0-(-3)}$ quotient law

$= 5^3$

Example 5 requires your knowledge on the rules of directed numbers. You can see that we are required to go through some steps before arriving at an answer, The negative indices rule makes it even easier. The following table shows indices written as negative and positive.

Negative Indices	**Positive Indices**
$\frac{1}{k^{-4}}$	k^4
$\frac{1}{x^{-2}}$	x^2
a^{-3}	$\frac{1}{a^3}$
2^{-5}	$\frac{1}{2^5}$
$\left(\frac{1}{2}\right)^{-3}$	2^3
2^{-3}	$\left(\frac{1}{2}\right)^3$

Following the rule makes it easy to change positive indices to positive and vice versa.

The next examples show how we can simplify algebraic expressions using the combination of all the laws.

Examples

1) Simplify $c^2 \times c^5$ and write your answer as a negative power.

Solution: $c^2 \times c^5 = c^7$ Product law

$= \frac{1}{c^{-7}}$

2) Simplify $(a^2)^3$ and write your answer with a negative power.

Solution: $(a^2)^3 = a^6$ Power law

$= \frac{1}{a^{-6}}$

3) Write $p^{-3} q^2$ as a positive power product.

Solution: $p^{-3} q^2 = \frac{1}{p^3} \times q^2$ Change p^{-3} to a positive power then multiply out.

$= \frac{q^2}{p^3}$

In mathematics a simplified answer is always written as a positive power.

4) Simplify $(5^{-2})(0.4)^0$.

Solution: $5^{-2} \times 1$ Zero index law

$= \frac{1}{5^2} \times 1$

$= \frac{1}{5^2}$ Answer as a positive exponent

$= \frac{1}{25}$ Numerals can be evaluated

The value of 5^{-2} is $\frac{1}{25}$, this is also described as being in its simplest form.

5) Simplify $(5a^{-3} bc^{-4})^2$

Solution: $(5a^{-3} bc^{-4})^2$ Power Law

$= 5^2a^{-6}b^2c^{-8}$ Evaluate 5^2

$= 25\, a^{-6}\, b^2\, c^{-8}$ Change powers to positive

$= (25)\left(\frac{1}{a^6}\right)(b^2)\left(\frac{1}{c^8}\right)$ Multiply across

$= \frac{25b^2}{a^6c^8}$

6) $(a^{-2}b^{-3})(a^{-4}b^5) = a^{-2 + -4} \times b^{-3 +5}$ Product law

$= a^{-6} \times b^2$ Multiply

$= a^{-6}\, b^2$

$= \frac{b^2}{a^6}$

7) $\dfrac{x^7y^{-2}}{x^4y^2} = x^{7-4}y^{-2-2}$ Quotient law

$=x^3y^{-4}$

$= \dfrac{x^3}{y^4}$

3. $(3a^2)^{-2} = 3^{-2} \times (a^2)^{-2}$ Power law

$= 3^{-2} \times a^{-4}$

$= \dfrac{1}{3^2} \times \dfrac{1}{a^4}$

$= \dfrac{1}{9} \times \dfrac{1}{a^4}$

$= \dfrac{1}{9a^4}$

NOW DO PRACTICE EXERCISE 12

Practice Exercise 12

1. Write down the value of each of these

a) 3^{-1} b) 5^{-2}

2. Write each with a negative index

a) $\frac{1}{11}$ b) $\frac{1}{5^4}$ c) $(a^{-5})(a^8)$

3. Simplify

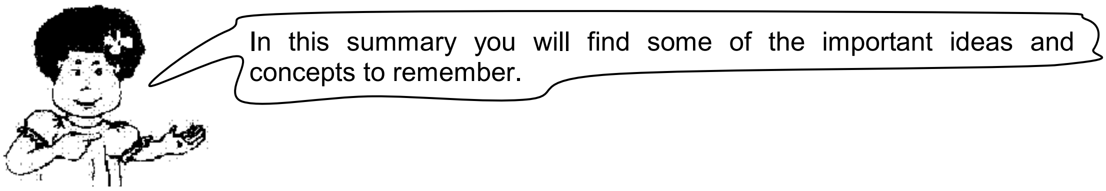
a) $(s^{-6}y^4)(s^2y^{-2})$ b) $(a^{-3}\,b^{-5})(a^5b^{-3})$

c) $\left(\frac{a^3}{b^2}\right)^{-2}$ d) $\left(\frac{3a^{-2}}{b^3}\right)^3$

e) $\left(\frac{5g^2}{h^{-3}}\right)^1$

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 2

SUB-STRAND 2: SUMMARY



- Coefficients are numbers in front of pronumerals.
- Exponent and index mean the same thing. Exponents show how many times a number is multiplied.
- The base is the number to be multiplied.
- The Index notation is the short form of writing long expressions.
- The product law states that when multiplying expressions of the same base you add the indices.
- Indices are the plural for index.
- When a power is raised to another power multiply the indices.
- When dividing index expressions with the same base subtract the indices.
- A number or expression raised to zero is equal to one.
- Indices can be written as positive or negative, following the rule

$$a^{-m} = \frac{1}{a^m} \text{ where } a \neq 0$$

- Terms or expressions are said to be in their simplest form when expressed with positive indices.

REVISE LESSONS 6 – 12 THEN DO SUB-STRAND TEST 2 IN ASSIGNMENT 6

ANSWERS TO PRACTICE EXERCISE 6 – 12

Practice Exercise 6

1.	(a)	$2^3 \times 3^2$	(b)	$6b^2$			
	(c)	$4y^3$	(d)	a^2b^4			

2. Coefficient of x^2 is 3 and coefficient of x is 2.

3.	(a)	8	(b)	81	(c)	8	
4.	(a)	$x \bullet x \bullet x \bullet x \bullet x$	b)	$5 \bullet 5 \bullet 5$			
	(c)	$5 \bullet 5 \bullet 5 \bullet a \bullet a \bullet a$	(d)	$5 \bullet a \bullet a \bullet a$			
5.	(a)	3	(b)	81 is the power of 3			
	(c)	4					
6.	(a)	$a^5 = a . a . a . a . a$	(b)	$a \bullet a \bullet a \bullet b \bullet b \bullet b$			
	(c)	$a \bullet a \bullet a \bullet a \bullet a \bullet a$					

Practice Exercise 7

1	(a)	a^5	(b)	b^7	(c)	m^8
	(d)	y^4	(e)	y^6	(f)	a^8
2.	(a)	$15a^3$	(b)	$30y^7$	(c)	$30x^4$
	(d)	$12b^4$	(e)	$6a^7$	(f)	$12b^6$
3.	(a)	$2a^2b^2$	(b)	x^3y^3	(c)	$6a^6$
	(d)	$6b^4$	(e)	$7x^6a^2$		

Practice Exercise 8

1.	(a)	b^{14}	(b)	p^{24}		
2.	(a)	a^7	(b)	b^{11}	(c)	c^8
3.	(a)	2^{12}	(b)	3^7		
4.	(a)	m^6	(b)	m^{10}	(c)	y^{12}
5.	(a)	m^4n^4	(b)	c^7d^7	(c)	$x^{15}y^6$

5. (d) $g^{10}h^5$ (e) $512p^9$ (f) $9j^{10}$
(g) $15\,625w^6$ (h) $512f^9$

6. (a) 76 (b) $2m^6$ (c) $4a^4$

Practice Exercise 9

1. (a) 3^4 (b) 5^1 (c) 10^2
(d) 8^1 (e) 4^6 (f) 2^7
(g) 5^4 (h) 10^6 (i) a^1
(j) y^6 (k) m^4 (l) x^6
(m) t^{-4} (n) a^{-7} (o) x^{-5}

2. (a) $2r^2$ (b) $\frac{1}{2}p^3$ (c) $3r^2$
(d) ab^2c^2 (e) $6r^7$ (f) $2p^7$

Practice Exercise 10

1. (a) a^{16} (b) a^9 (c) $27a^7b^{12}$
(d) $81s^9y^9$

2. (a) $\frac{3}{5}x^3$ (b) $\frac{8}{21}a^2$ (c) $4ab^6c^3$
(d) a^6bc^5

3. (a) a^{12} (b) m^{21} (c) $4x^8$
(d) $81x^6$ (e) $256x^8y^{12}$ (f) $27m^6n^6p^9$
(g) $\frac{x^2}{y^2}$ (h) $\frac{y^2}{z^2}$ (i) $27\frac{x^{12}}{y^6}$
(j) $1024\frac{m^{10}}{n^{10}}$ (k) $\frac{6561a^8b^{12}}{c^{12}}$ (l) $\frac{49m^4n^6}{p^{10}}$

Practice Exercise 11

1. (a) 1 (b) 1 (c) 1
(d) 1 (e) 7 (f) 3
(g) 8 (h) 4 (i) b^2
(j) n^4 (k) c^3 (l) x^7
(m) k (n) a (o) $-3a^4$
(p) $-8g^{11}$

2. (a) Both are equal. (b) $7k^0$ is larger.

Practice Exercise 12

1. (a) $\frac{1}{3}$ (b) $\frac{1}{25}$

2. (a) 11^{-1} (b) 5^{-4} (c) $\frac{1}{a^{-3}}$

3. (a) $\frac{y^2}{s^4}$ (b) $\frac{a^2}{b^8}$ (c) $\frac{b^4}{a^6}$

   (d) $\frac{27}{a^6b^3}$ (e) $5g^2h^3$

END OF SUB STRAND 2

SUB-STRAND 3

POLYNOMIALS

SUB-STRAND 3: POLYNOMIALS

Introduction

In this sub strand we will study algebraic expressions called polynomials.

Polynomials have the same importance in algebra that whole numbers have in arithmetic.

Most of the work in arithmetic involves operations with whole numbers.

In the same way, much of the work in algebra involves operations with polynomials.

We will discuss operations with polynomials in this sub-strand.

EXAMPLES OF POLYNOMIALS WITH SPECIAL NAMES

(i)	$5x^2y$	-	**Monomial (one term)**
(ii)	$-3xy^3 + 2x^2y$	-	**Binomial (two unlike terms)**
(iii)	$4x^2y^2 – 7xy + 6$	-	**Trinomial (three unlike terms)**

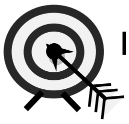
Note: We will not use special names for polynomials with more than three terms

In this sub strand you will:

- classify polynomials according to the number of terms
- add and subtract polynomials either vertically or horizontally and
- multiply and divide polynomials.

Lesson 13: Classification of Polynomials

In the previous lesson you have learnt about writing index expressions with a positive or a negative power.



In this lesson, you will:

- define and identify Polynomials
- classify polynomials according to the number of terms.
- determine the degree of a polynomial.

Let us define the term **POLYNOMIAL.**

A polynomial is an algebraic expression, in which the variable x does not appear in the denominator of any term.

For example:

These are polynomials

1) $x + 3$

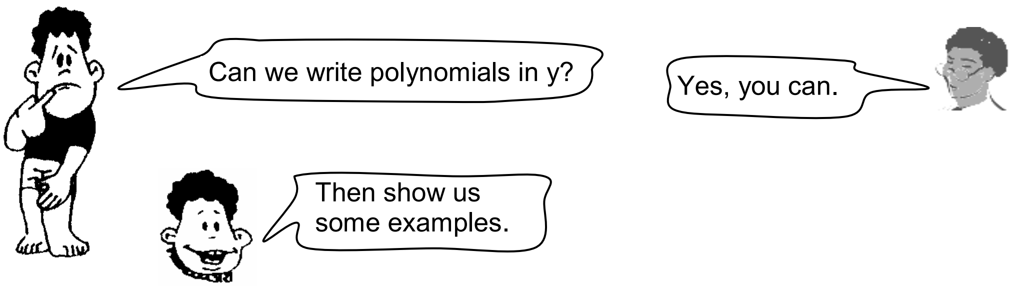
2) $x^2y - 2xy + x$

3) $\frac{3}{4}x - 7$

These are not polynomials

1) $\frac{1}{x} + 1$

2) $x + 2 - \frac{1}{x^2 - y}$



Here are some examples of polynomials in y. You can use any letter.

1) $2y + 1$ 2) $\frac{y}{2} - 1$ 3) $ay^2 + by + c$

The following are **not** polynomials in y.

1) $\frac{1}{y} - 2$ 2) $\frac{2}{y^2 - 3} + 5$ 3) $1 - \frac{11}{y^2 - x^2}$

They are not polynomials simply because there are y in the denominator.

Polynomials are algebraic expressions. Like any other expression, polynomials are made up of terms separated by a plus (+) or a minus (-) sign. Polynomials can be named according to the number of terms it has. The three types of polynomials we will look at are:

1) Monomial
2) Binomial
3) Trinomial

- A polynomial of one term is called a **monomial.**
- A polynomial of two unlike terms is called a **binomial.**
- A polynomial of three unlike terms is called a **trinomial**.

Here is a table showing examples of the types of polynomials.

Monomial	**Binomial**	**Trinomial**
5	2s - 3	$6z^2 - 3z + 1$
$2x^2$	x - 5	$4p^2 - 12p + 9$
$5a^3bc^2$	$5y^2 + 3y$	$x^2 - 2x - 1$
$\frac{2}{3}x$	$\frac{1}{2}p - 7$	a – b +c

Polynomials are usually written this way, with the terms written in "decreasing" order; that is, with the largest exponent first, the next highest next, and so forth, until you get down to the constant.

Leading term

Constant

$$\mathbf{4x^2 + 7x - 2}$$

Terms

The first term in the polynomial, when it is written in decreasing order, is also the term with the biggest exponent, and is called the **"leading term".**

The exponent on a term tells you the "degree" of the term. For instance, the leading term in the above polynomial is a "**second-degree term**" or "**a term of degree two**". The second term is a "**first degree**" term. The degree of the leading term tells you the degree of the whole polynomial. The polynomial above is a "second-degree polynomial". Here are a few more examples.

1) The trinomial $2x^2 – 5x + 9$, has three terms. Included are: a second degree term, a first degree term and a constant. This is a **second degree polynomial.**

2) The trinomial $7x^4 + 6x^2 + x$, has a fourth degree term, second degree term and a first degree term. There is no constant. This is a **fourth degree polynomial.**

3) The monomial $25x^3$ is a **third degree polynomial.**

4) The trinomial $9x^3 – 7x + 5$ is a **third degree polynomial.**

5) $x^5 – 7x^4 – 3x^2 + 21x$ is called a **fifth degree polynomial.**

NOW DO PRACTICE EXERCISE 13

Practice Exercise 13

1 Classify as monomial, binomial or trinomial.

a) $2x^2 + \frac{1}{3}x$ b) $\frac{1}{9}y^2z - 5$ c) $x^2 + 5x + 4$

d) $2y + 6$ e) $x^2 + x + 10$ f) $\frac{x}{3} + 10$

g) $x^3y^3 - 3^2x^2y + 2$ h) $\frac{1}{2x^2 - 5}$

---

2. Write each polynomial in descending powers of the indicated letter.

a) $7x^3 - 4x - 5 + 8x^5$ Powers of x

b) $10 - 3y^5 + 4y^2 - 2y^3$ Powers of y

c) $3x^2y + 8x^3 + y^3 - y^5$ Powers of y

d) $6y^3 + 7x^2y - 4y^2 + y$ Powers of y

---

3. Refer to $5x^4 + x^3 - x^2 + 1$ to answer the following questions.

a) How many terms are in the polynomial?

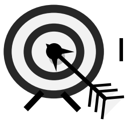
b) Name the leading term.

c) Name the degree of the polynomial.

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 3

Lesson 14: Adding and Subtracting Polynomials

In the previous lesson you have defined and identified Polynomials, classified polynomials according to the number of terms and determined the degree of polynomials.



In this lesson, you will:

- arrange polynomials in descending powers
- add and subtract polynomials either vertically or horizontally.

In the last lesson you learnt how to write polynomials in descending order with the largest exponent first, followed by the next largest and so forth. Remember that polynomials are always written in descending order.

Examples

1) $4x + 2x^2 + 2$ can be written as $2x^2 + 4x + 2$

2) $2y^3 – 2 + 5y^4$ can be written as $5y^4 + 2y^3 – 2$

In sub-strand 2 of this strand you learnt how to simplify algebraic expressions by collecting like terms. Polynomials are also algebraic expressions. Therefore the practice of collecting like terms also applies to adding and subtraction of polynomials. This lesson will also require you to recall and use the rules of the order of operations, directed numbers and other algebraic laws.

When we add or subtract polynomials horizontally we group the like terms (of the same degree) together and add or subtract them by collecting like terms.

Examples

1 $2x + 3 + 4x – 1$

$= 2x + 3 + 4x – 1$ collect and group like terms

$= (2x + 4x) + (3 – 1)$ add coefficients of like terms

$= 6x + 2$

2. $(3x^2 + x – 2) + 2x^2$ remove grouping symbol

$= 3x^2 + x – 2 + 2x^2$ collect and group like terms

$= (3x^2 + 2x^2) + x – 2$ add coefficients of like terms

$= 5x^2 + x – 2$

3. $(3x^2 + 5x – 4) + (2x + 5) + (x^3 – 4x^2 + x)$

$= 3x^2 + 5x – 4 + 2x + 5 + x^3 - 4x^2 + x$

$= x^3 + (3x^2 - 4x^2) + (5x + 2x + x) + (- 4 + 5)$

$= x^3 – x^2 + 8x + 1$

The third example looks challenging but it can turn out very easy once you practice doing the example yourself many times.

4. $(5x^3y^2 – 3x^2y^2 + 4xy^3) + (4x^2y^2 – 2xy^2) + -7x^3y^2 + 6xy^2 – 3xy^3)$

$= \quad 5x^3y^2 – 7x^3y^2 – 3x^2y^2 + 4x^2y^2 + 4xy^3 – 3xy^3 – 2xy^2 + 6xy^2$

$= \quad – 2x^3y^3 + x^2y^2 + xy^3 + 4xy^2$

Most addition of polynomials will be done horizontally as already shown, but in a few cases it is convenient to use vertical addition.

- **When adding vertically arrange the polynomials in descending order.**
- **Arrange the terms under one another so that the like terms are in the same vertical line.**
- **Find the sum of the terms in each vertical line by adding the numerical coefficients.**

Example 1

Add (2x + 5y) + (3x – 2y)

Solution:

$$\begin{array}{r} 2x + 5y \\ \underline{3x - 2y} \\ 5x + 3y \end{array}$$

Add each column down

Example 2

Add the Polynomials $7x^2 + 3x – 4$ and $x^2 –x + 5$

Solution:

$$\begin{array}{r} 7x^2 + 3x - 4 \\ \underline{x^2 - x + 5} \\ \underline{8x^2 + 2x + 1} \end{array}$$

Add each column down

Example 3

$(3x^2 + 2x – 1) + (2x + 5) + (4x^3 + 7x^2 – 6)$

$$\begin{array}{r} 4x^3 + 7x^2 + 0x - 6 \\ 3x^2 + 2x - 1 \\ \underline{2x + 5} \\ 4x^3 + 10x^2 + 4x - 2 \end{array}$$

Add each column down

Notice that in the first row $4x^3 + 7x^2 - 6$ is written as $4x^3 + 7x^2 + \mathbf{0x} - 6$, this is because the polynomial does not have a first degree term hence 0x is used instead to fill in the column. You can do the same to any polynomial with a missing term of degree.

SUBTRACTION

Polynomials can be subtracted horizontally by collecting like terms.

To subtract polynomials change the sign of all the terms in the subtrahend (polynomial being subtracted) then proceed to addition of polynomials.

Example 1

Subtract $(4x + 6) - (x + 2)$.

Solution: $(4x + 6) - (x + 2)$ remove the grouping symbol

$= 4x + 6 - x - 2$ change the signs

$= 4x - x + 6 - 2$ the sign in front of the term moves with the term, then subtract.

$\mathbf{= 3x + 4}$ **Answer**

Example 2

Subtract: $(4x + 6) - (x - 2)$

Solution: $(4x + 6) - (x - 2)$ remove the grouping symbol

$= 4x + 6 - x + 2$ change the signs

$= 4x - x + 6 + 2$ add or subtract

$\mathbf{= 3x + 8}$ **Answer**

Example 3

Subtract $3x^2 - 4x + 1$ from $5x^2 - x + 5$

$= (5x^2 - x + 5) - (3x^2 - 4x + 1)$

$= 5x^2 - x + 5 - 3x^2 + 4x - 1$

$= 5x^2 - 3x^2 - x + 4x + 5 - 1$

$\mathbf{= 2x^2 + 3x + 4}$ **Answer**

The next few examples are challenging but are easy if you practise the question a number of times.

Example 4

Subtract $(-4x^2y + 10xy^2 + 9xy-7)$ from $(11x^2y – 8xy^2 + 7xy + 2)$

Solution:

$(11x^2y – 8xy^2 + 7xy + 2) – (-4x^2y + 10xy^2 + 9xy-7)$

$=11x^2y – 8xy^2 + 7xy + 2 + 4x^2y – 10xy^2 – 9xy + 7$ collect like terms

$=11x^2y + 4x^2y – 8xy^2 – 10xy^2 + 7xy – 9xy + 2 + 7$ add the terms

$=\mathbf{15x^2y – 18xy^2 – 2xy + 9}$ **Answer**

Example 5

Subtract $(4x^2y^2 + 7x^2y – 2xy + 9)$ from $(6x^2y^2 – 2x^2y + 5xy + 8)$

Solution:

$(6x^2y^2 – 2x^2y + 5xy + 8) – (4x^2y^2 + 7x^2y – 2xy + 9)$

$= 6x^2y^2 – 2x^2y + 5xy + 8 – 4x^2y^2 – 7x^2y + 2xy – 9$

$= 6x^2y^2– 4x^2y^2– 2x^2y – 7x^2y + 5xy + 2xy + 8 – 9$

$\mathbf{= 2x^2y^2 – 9x^2y + 7xy –1}$ **Answer**

Most subtraction of polynomials will be done horizontally as already shown, but in a few cases it is convenient to use vertical subtraction.

To Subtract Polynomials Vertically

1. **Write the polynomial being subtracted under the polynomial from which it is being subtracted. Write like terms in the same vertical line.**
2. **Mentally change the sign of each term in the polynomials being subtracted**
3. **Find the sum of the resulting terms in each vertical line by adding their numerical coefficients.**

Example 7

Subtract $2x – 3y$ from $10x + 5y$

$$\begin{array}{r} 10x + 5y \\ \underline{-2x + 3y} \\ \mathbf{8x + 8y} \end{array}$$

change signs, then add vertically

Example 8

$(10x^2 + 2x - 5) - (5x^2 + x + 4)$

$$\begin{array}{r} 10x^2 + 2x - 5 \\ \underline{-5x^2 - x - 4} \\ \mathbf{5x^2 + x - 9} \end{array}$$

Example 9

Subtract $(4x^2y^2 + 7x^2y - 2xy + 9)$ from $(6x^2y^2 - 2x^2y + 5xy + 8)$

$$\begin{array}{r} 6x^2y^2 - 2x^2y + 5xy + 8 \\ \underline{-4x^2y^2 - 7x^2y + 2xy - 9} \\ \mathbf{2x^2y^2 - 9x^2y + 7xy - 1} \end{array}$$

Example 10

Subtract $3x^2 - 4x + 1$ from $5x^2 - x + 3$

$$\begin{array}{r} 5x^2 - x + 3 \\ \underline{-3x^2 + 4x - 1} \\ \mathbf{2x^2 + 3x + 2} \end{array}$$

NOW DO PRACTICE EXERCISE 14

Practice Exercise 14

1. Add vertically.

   a) $(14x + 5) + (10x + 5)$

   b) $(10x + 12) + (6x + 20)$

   c) $(19x^2 + 12x + 12) + (7x^2 + 10x + 13)$

   d) $(17x^2 + 20x + 11) + (15x^2 + 11x + 17)$

---

2. Add horizontally.

   a) $(14x + 5) + (10x + 5)$

   b) $(10x + 12) + (6x + 20)$

   c) $(19x^2 + 12x + 12) + (7x^2 + 10x + 13)$

   (d) $17x^2 + 20x + 11) + (15x^2 + 11x + 17)$

---

3. Compare your answers to questions one and two.

---

4. Subtract horizontally.

   a) $(6x + 14) – (9x + 5)$

   b) $(6x + 19) – (14x + 5)$

   c) $(14x^2 + 13x + 12) – (7x^2 + 20x + 4)$

   d) $(-9x^2 – 4x – 4) – (-9x^2 – 11x + 12)$

---

5. Subtract vertically.

   a) $6x + 14) – (9x + 5)$

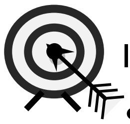
   b) $(6x + 19) – (14x + 5)$

   c) $(14x^2 + 13x + 12) – (7x^2 + 20x + 4)$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 3

Lesson 15: Multiplying a Polynomial by a Monomial

In the previous lesson you arranged polynomials in descending powers and added and subtracted polynomials either vertically or horizontally.



In this lesson, you will:

- apply the laws of exponents in multiplying a polynomial by a monomial.
- multiply a polynomial by a monomial.

You have learnt how to add and subtract polynomials. Adding and subtracting polynomials horizontally require you to collect like terms, then add and subtract. You also learnt how to add and subtract polynomials vertically. Whatever the case maybe, you still need to use the rules and laws of algebra.

You will also need the same rules and laws of algebra in this lesson. .

How do you multiply a polynomial by monomial?

Use the distributive law to multiply the monomial by every term in the polynomial.
The distributive law states $a(b + c) = ab + ac$

Examples

1) $4(2x + 3) = 8x + 12$

2) $4(2x – 3) = 8x – 12$

3) $-2(2a + 8) = -4a – 16$

4) $cd(6c^2 + 3cd) = 6c^3d + 3c^2d^2$

5) $4z^2(z^2 + 7z – 3z^2) = 4z^4 + 28z^3 – 12z^4$
$= (4z^4– 12z^4) + 28z^3$ Collect like terms
$= -8z^4 + 28 z^3$

In example 5, notice that we needed the product law, rules on directed numbers as well collecting like terms.

6) Multiply $5x$ and $3x^2 - 2x + 6$

$5x (3x^2 - 2x + 6) = 15x^3 – 10x^2 + 30x$

With such questions as example 6 put the polynomial inside parentheses and multiply the monomial by every term of the polynomial.

8) $(-5ab^2)(2a^3bc^2 – 3ac + b^3)$

$= -10a^4b^3c^2 + 15a^2b^2c – 5ab^5$

9) $3x(4x^2 + 7x - 5)$

$= (3x) \times (4x^2) + (3x) \times (7x) + (3x) \times (-5)$

$= 12x^3 + 21x^2 – 15x$

10) Find the product of $2n(9n^2 – 2n – 12)$.

Solution: $2n(9n^2 – 2n – 12) = 18n^3 – 4n^2 – 24n$

12. Simplify $\frac{1}{2}u(14u – 2) + 25u$

Solution: $\frac{1}{2}u(14u – 2) + 25u$

$= 7u^2 – u + 25u$ Collect like terms
$= 7u^2 + 24u$

NOW DO PRACTICE EXERCISE 15

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Practice Exercise 15

1. Find the product

(a) $6(a + 2)$ (b) $2(p – q)$

(c) $s(c + 2)$ (d) $(p – 4)3$

2. Simplify

(a) $2(x + 5) + 8$ (b) $3a + 2(a + b)$

(c) $6(x + y) + 2y$ (d) $6a + 2(a + 4)$

(e) $3(4x + 2) – 4$ (f) $3(2x + 2) - 4x$

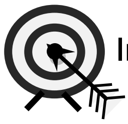
g) $6x(5x^2 – 3x – 7)$ h) $8y (3y^2 + 4y – 9)$

i) $(2m^3 + m^2 – 6m) (-4m^2)$ j) $(7h^3 – 5h – 11) (-3h^3)$

CHECK YOUR WORK. ANSWERS ARE AT THE END OF THE SUB-STRAND 3

Lesson 16: Multiplying a Polynomial by another Polynomial

In the previous lesson you learnt how to multiply a polynomial by a monomial.



In this lesson, you will:

- identify methods of multiplying a polynomial by another polynomial
- actually multiply a polynomial by another polynomial.

You learnt the distributive law and other algebraic rules in sub-strand 1. These laws will be useful in this lesson for multiplying a polynomial by another polynomial. The distributive law mathematically is stated as $a(b + c) = ab + ac$.

Here is a diagram showing what we mean by distribute.

$$\mathbf{a\ (\ b + c\ ) = ab + ac}$$

Notice that **a** has been distributed across the addition to b and c. The distributive law also works with subtraction $a\ (\ b – c\ ) = ab – ac$.

In the previous lesson we used the distributive law to multiply a monomial by a polynomial. This lesson will require you to use the distributive law to multiply a polynomial by a polynomial.

There are two methods which you can use to multiply a polynomial by another polynomial. You can multiply horizontally or vertically, in both methods the distributive law is used.

Let us first multiply polynomials horizontally.

Examples

1) $2x\ (x + y) = 2x^2 + 2xy$

2) $-2a(5ab + 3a^2b^2 + 7a^3b^3) = -10a^2b – 6a^3b^2 – 14a^4b^3$

In the two examples above the term outside is multiplied by every term in the parentheses using the distributive law.

This can be expanded to multiplying a binomial by a binomial. Binomials have two terms. To multiply a binomial by a binomial, you will distribute both terms of one polynomial times both terms of the other polynomial.

One way to keep track of your distributive property is to use the **FOIL method**.

FOIL stands for **first terms**, **outside terms**, **inside terms** and **last terms**. In other words do the distributive property for every term in the first binomial. That is multiply each term in the first binomial by every term on the second binomial. Here is a diagram showing the explanation.

Inside terms Last terms

$$(a + b)(c + d) = ac + ad + bc + bd$$

First terms Outside terms

The examples below further explain the diagram.

Examples

3) $(x + 5)(x + 2) = x(x + 2) + 5(x + 2)$ Distributive law

$= x^2 + 2x + 5x + 10$ Collect like terms

$= x^2 + 7x + 10$

4) $(3x + 5)(2x – 7) = 3x(2x – 7) + 5(2x – 7)$ Distributive law

$= 6x^2 – 21x + 10x – 35$ Collect like terms

$= 6x^2 – 11x – 35$

5) $(6x + 3)(-5x + 2) = 6x(-5x + 2) + 3(-5x + 2)$ Distributive law

$= -30x^2 + 12x – 15x + 6$ Collect like terms

$= -30x^2 – 3x + 6$

To multiply a multi term polynomial by a multi term polynomial requires the use of distributive law as well.

Examples

6) $(x + 3)(4x^2 – 4x – 7) = x(4x^2 – 4x – 7) + 3(4x^2 – 4x – 7)$ Distributive

$= 4x^3 – 4x^2 – 7x + 12x^2 – 12x – 21$ Like terms

$= 4x^3 + 8x^2 – 19x – 21$

7) $(-3x + 8)(-4x^2 + 2x + 5) = -3x(-4x^2 + 2x + 5) + 8(-4x^2 + 2x + 5)$

$= 12x^3 – 6x^2 –15x – 32x^2 + 16x + 40$

$= 12x^3 – 38x^2 + x + 40$

8) $(x + y + z)(2x + 1) = x(2x + 1) + y(2x + 1) + z(2x + 1)$

$= 2x^2 + x + 2xy + y + 2xz + z$

Calculating bigger multiplications of polynomials vertically is faster and simpler than multiplying horizontally. Multiplying vertically can be done in the same manner as multiplying numerals as this.

$$\begin{array}{r} 1\,2\,1 \\ \times\ \ 3\,2 \\ \hline 2\,4\,2 \\ 3\,6\,3\ \ \\ \hline 3\,8\,7\,2 \\ \hline \end{array}$$

Here are the steps to follow when multiplying algebraic expressions.

1. Arrange the polynomials vertically.
2. Take one term at a time from the lower polynomial and multiply each term of the upper polynomial, work from the right to the left.
3. The Product of each lower polynomial forms a new row. Like terms must be lined up. Add each column to give the result.

Examples

1)

$$\begin{array}{r} x + y \\ 2x \\ \hline 2x^2 + 2xy \\ \hline \end{array}$$

2)

$$\begin{array}{rr} & x + 3 \\ & x - 2 \\ \hline & -2x - 6 \\ x^2 + 3x & \\ \hline x^2 + x - 6 & \\ \hline \end{array}$$

Add each column

product of $-2(x + 3)$

product of $x(x + 3)$

3) Find the product of $3x + 5$ and $2x - 7$

$$\begin{array}{rr} & 3x + 5 \\ & 2x - 7 \\ \hline & -21x - 35 \\ 6x^2 + 10x & \\ \hline 6x^2 - 11x - 35 & \\ \hline \end{array}$$

In example 1 and 2, the like terms of a product were lined up. This makes it easy to find the sum of columns to get the final product of the two polynomials.

4)

$$\begin{array}{r} x^2 - 3x + 2 \\ \underline{x - 5} \\ -5x^2 + 15x - 10 \\ \underline{x^3 - 3x^2 + 2x \quad\quad} \\ \underline{x^3 - 8x^2 + 17x - 10} \end{array}$$

5) Multiply $x + 3$ by $4x^2 - 4x - 7$

$$\begin{array}{r} 4x^2 - 4x - 7 \\ \underline{x + 3} \\ 12x^2 - 12x - 21 \\ \underline{4x^3 - 4x^2 - 7x \quad\quad} \\ \underline{4x^3 + 8x^2 - 19x - 21} \end{array}$$

NOW DO PRACTICE EXERCISE 16

Practice Exercise 16

1. Use FOIL to calculate.

a) $(m - 3)(m + 3)$ b) $(x - 5)(x + 2)$

c) $(y + 7)(y - 3)$ d) $(3m - 2)(m - 1)$

2. Multiply $(-2x^2 + 4x + 11)(5x + 12)$ vertically.

3. Multiply $(-11x + 3)(10x^2 - 7x + 9)$ horizontally.

4 Find the product.

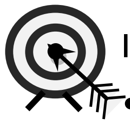
a) $(4 + a^4 + 3a^2 - 2a)(a + 3)$

b) $(3b - 5 + b^4 - 2b^3)(b - 5)$

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 3

Lesson 17: Dividing a Polynomial by a Monomial

In the previous lesson you have identified methods of multiplying a polynomial by another polynomial and then multiplied a polynomial by another polynomial.



In this lesson, you will:

- apply the Quotient Law in dividing a polynomial by a monomial
- divide a polynomial by a monomial

You learnt the Quotient law in sub-strand 2. The quotient law states that, when dividing exponents of the same base subtract the indices.

Example $\frac{x^7}{x^3} = x^4$

You also learnt in grade 7, that the quotient of two integers with the same sign is a positive number, if the two integers have unlike signs then the quotient will be negative that is -8 ÷ 2 = -4 or -8 ÷ -2 = 4. In the same manner, the rule can be applied to polynomials.

When dividing a Polynomial by a Monomial:

- **Divide like terms.**
- **Use the Quotient law if the polynomial has exponents of the same base.**
- **If the divisor and the dividend have the same sign then the Quotient will be positive.**
- **If the divisor and the dividend have unlike signs then the Quotient will be negative.**
- **Use other algebraic properties to simplify the Quotient.**

Example 1

Divide $15x^2 – 9$ by 3

Solution: $(15x^2 - 9) \div 3 = \frac{15x^2 - 9}{3}$

$= \frac{15x^2}{3} - \frac{9}{3}$

$= 5x^2 - 3$

Example 2

Divide $25y^2 - 20$ by 5

Solution $25y^2 -20 \div 5 = \dfrac{25y^2 - 20}{5}$

$$= \frac{25y^2}{5} - \frac{20}{5}$$

$$= 5y^2 - 4$$

Example 3

Divide $8x^3 + 2x^2$ by $2x$.

Solution $(8x^3 + 2x^2) \div 2x$

$= \dfrac{8x^3}{2x} + \dfrac{2x^2}{2x}$ Divide like terms

$= 4x^{3-1} + 1x^{2-1}$ Quotient law

$= 4x^2 + x$

Example 4 $\dfrac{14a^3b - 21a^2b + 28ab}{7ab}$

$= \dfrac{14a^3b}{7ab} - \dfrac{21a^2b}{7ab} + \dfrac{28ab}{7ab}$ Divide like terms

$= 2a^{3-1}b^{1-1} - 3a^{2-1}b^{1-1} + 4$ Quotient law

$= 2a^2b^0 - 3ab^0 + 4$

$= 2a^2 - 3a + 4$

Example 5 $\dfrac{x^3 - x}{x} = \dfrac{x^3}{x} - \dfrac{x}{x}$

$= x^{3-1} - x^{1-1}$ Quotient law

$= x^2 - x^0$

$= x^2 - 1$

NOW DO PRACTICE EXERCISE 17

Practice Exercise 17

Divide the following:

1) $\dfrac{9x^3 - 3x}{x}$

2) $\dfrac{8a^9 - 4a^4}{4a^4}$

3) $\dfrac{27x^{10} - 15x^5}{x^5}$

4) $\dfrac{n^3 - 3n^2}{n^2}$

5) $\dfrac{4x^2 + x^3}{x^2}$

6) $\dfrac{14a^5 + 2a^2}{2a^2}$

7) $\dfrac{18a^4 - 12a^2}{3a^2}$

8) $\dfrac{100y^2 - 10y^5}{10y^3}$

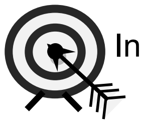
9) $\dfrac{3rs - 3rs + 15r}{3r}$

10) $\dfrac{-2b^2 + 2b^2 + 8b}{2b}$

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 4

Lesson 18: Dividing a Polynomial by a Polynomial

In the previous lesson you have applied the Quotient Law in dividing a polynomial by a monomial.



In this lesson, you will:

- apply the Quotient Law in dividing a polynomial by another polynomial
- actually divide a polynomial by another polynomial

In the previous lesson, we discussed the use of the Quotient law in dividing a polynomial by a monomial.

Examples

1) $\dfrac{28x^2}{7x} = 4x$ by the quotient law

2) $(14x^4 + 10x^2 + 6x) \div 6x = \dfrac{12x^4 + 18x^2 + 6x}{6x}$ by the quotient law

$= 2x^3 + 3x + 1$

3) $(15x^3 + 20x^2 + 5x) \div 5 = \dfrac{15x^3 + 20x^2 + 5x}{5}$ by the quotient law

$= 3x^3 + 4x^2 + x$

Dividing a polynomial by something more complicated than a simple monomial will require the need to use a different method to simplify. This method is called the long division method. The long division method works the same way as the long division of numerals you did back in lower primary school except that long division of polynomials involves variables.

In Lower Primary, you learnt that, 966 ÷ 23 =

$$
\begin{array}{r}
42 \\
23\overline{)966} \\
-92 \\
\hline
46 \\
-46 \\
\hline
0
\end{array}
$$

From the long division procedure shown above, 42 is called the Quotient, 23 is the divisor and 966 is the dividend and 0 is the remainder. Hope you remember this from grade 5.

To divide a polynomial by long division:

- **Set up the division putting both polynomials in standard form that is descending power of the variables.**
- **Divide first term of the dividend by the first term of the divisor and write the result over the dividend**
- **Multiply the result in step 2 by each term in the divisor and write the result under the dividend**
- **Subtract the result in step 3 from the dividend**
- **Bring the next term down**
- **Repeat step 2 through to step 5 until there are no other terms to bring down. If you have a remainder then place it over the divisor.**

Example 1 $(x^2 - 3x - 10) \div (x + 2)$

Solution:

$$
\begin{array}{r l}
 & x - 5 \\
x + 2 \,\big) & x^2 - 3x - 10 \\
 & x^2 + 2x \\
\hline
 & \quad -5x - 10 \\
 & \quad -5x - 10 \\
\hline
 & \qquad\qquad 0
\end{array}
$$

$(x^2 - 3x - 10) \div (x + 2) = x - 5$

Example 2 Divide $12x^3 - 9x^2 + 13x - 12$ by $x - 3$

Solution:

$$
\begin{array}{r l}
 & 2x^2 - 3x + 4 \\
x - 3 \,\big) & 2x^3 - 9x^2 + 13x - 12 \\
 & 2x^3 - 6x^2 \\
\hline
 & \quad -3x^2 + 13x - 12 \\
 & \quad -3x^2 + 9x \\
\hline
 & \qquad\qquad 4x - 12 \\
 & \qquad\qquad 4x - 12 \\
\hline
 & \qquad\qquad\qquad 0
\end{array}
$$

When the divisor is a polynomial of more than two terms, exactly the same procedure is used.

Example 3 Divide $x^3 -4x^2 + x + 6$ by $x+1$

Solution:

$$\begin{array}{r|l} & x^2 - 5x + 6 \\ \hline x+1 & x^3 - 4x^2 + x + 6 \\ & \underline{x^3 + x^2} \\ & \quad -5x^2 + x + 6 \\ & \quad \underline{-5x^2 - 5x} \\ & \qquad 6x + 6 \\ & \qquad \underline{6x + 6} \\ & \qquad\quad 0 \end{array}$$

You can check whether your answer is correct or not by using the following rule:

Dividend = Quotient × Divisor

Let us check the answer to example 4

Quotient × Divisor = Dividend

$(x^2 - 5x + 6)(x + 1) = x^3 - 5x^2 + 6x + x^2 - 5x + 6$
$= x^3 - 4x^2 + x + 6.$

The product is equal to the dividend. The answer in example 5 is therefore correct.

You can check the other answers as well.

NOW DO PRACTICE EXERCISE 18

Practice Exercise 18

1. Simplify.

(a) $27x^2 \div 9$

(b) $27x^2 \div (-3)$

(c) $(40x^2 + 28) \div 4$

(d) $(6p - 3) \div (-3)$

(e) $(-12y^5 - 10x^3) \div (-2)$

(f) $(18a^2 - 12b^2 + 6) \div 6$

---

2. Divide the following and check your answer.

(a) $y^{10} \div y^8$

(b) $x^9 \div (-x^3)$

(c) $20z^4 \div 10z$

(d) $2f^4 \div (-3f^2)$

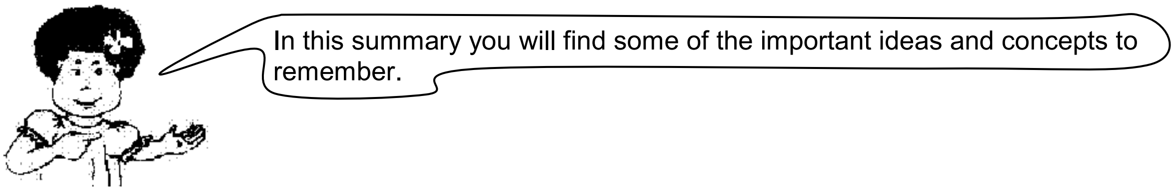
(e) $\dfrac{4r^2 - 8}{2}$

(f) $\dfrac{12r^4 - 3r^3}{r^3}$

(g) $\dfrac{27r^4 - 12r^2 + 9r}{-3r}$

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND

SUB-STRAND 3: SUMMARY



- Polynomials are algebraic Expressions, in which variables do not appear in the denominator of any term. There are different types of polynomials.
- A polynomial of one term is called a monomial.
- A polynomial of two unlike terms is called a binomial.
- A polynomial of three unlike terms is called a trinomial.
- Polynomials are usually written in descending powers of one of the letters.
- Polynomials can be added horizontally by combining like terms.
- To add polynomials horizontally, it is helpful to underline all like terms of the same degree with the same kind of line before adding.
- To add polynomials vertically, arrange them under one another so that like terms are in the same vertical line.
- Polynomials can be subtracted horizontally by combining like terms.
- Polynomials can be subtracted vertically.
- To multiply a polynomial by a monomial multiply each term in the polynomial by the monomial; then add the resulting products.
- To divide a polynomial by a monomial, divide each term in the polynomial by the monomial; then add the resulting quotients.

REVISE LESSONS 13-18. THEN DO SUB-STRAND TEST 3 IN ASSIGNMENT 6

ANSWERS TO PRACTICE EXERCISES 13 – 18

Practice Exercise 13

1. (a) binomial (b) binomial (c) trinomial (d) binomial
   (e) trinomial (f) binomial (g) trinomial (h) neither

2. (a) $8x^5 + 7x^3 - 4x - 5$
   (b) $-3y^5 - 2y^3 + 4y^2 + 10$
   (c) $-y^5 + y^3 + 3x^2y + 8x^3$
   (d) $6y^3 - 4y^2 + y + 7x^2y$

3. (a) 4 terms
   (b) $5x^4$
   (c) the 4th power.

Practice Exercise 14

1. (a)
$$\begin{array}{r} 14x + 5 \\ 10x + 5 \\ \hline 24x + 10 \\ \hline \end{array}$$
   (b)
$$\begin{array}{r} 10x + 12 \\ 6x + 20 \\ \hline 16x + 32 \\ \hline \end{array}$$
   (c)
$$\begin{array}{r} 19x^2 + 12x + 12 \\ 7x^2 + 10x + 13 \\ \hline 26x^2 + 22x + 25 \\ \hline \end{array}$$
   (d)
$$\begin{array}{r} 17x^2 + 20x + 11 \\ 15x^2 + 11x + 17 \\ \hline 32x^2 + 31x + 28 \\ \hline \end{array}$$

2. (a) $14x + 10x + 5 + 5 = 24x + 10$
   (b) $10x + 6x + 20 + 12 = 16x + 32$
   (c) $19x^2 + 7x^2 + 12x + 10x + 12 + 13 = 26x^2 + 22x + 25$
   (d) $17x^2 + 15x^2 + 20x + 11x + 11 + 17 = 32x^2 + 31x + 28$

4. (a) $6x - 9x + 14 - 5 = -3x + 9$
   (b) $6x - 14x + 19 - 5 = -8x + 14$
   (c) $14x^2 - 7x^2 + 13x - 20x + 12 - 4 = 7x^2 - 7x + 8$
   (d) $-9x^2 + 9x^2 - 4x + 11x - 12 = 7x - 16$

5. (a)
$$\begin{array}{r} 6x + 14 \\ -9x - 5 \\ \hline -3x + 9 \\ \hline \end{array}$$
   (b)
$$\begin{array}{r} 6x + 19 \\ -14x - 5 \\ \hline -8x + 14 \\ \hline \end{array}$$

c)

$$\begin{array}{r} 14x^2 + 13x + 12 \\ -7x^2 - 20x - 4 \\ \hline 7x^2 - 7x + 8 \\ \hline \end{array}$$

Practice Exercise 15

1. (a) $6a + 12$ (b) $2p - 2q$
   (c) $sc + 2s$ (d) $3p - 12$

2. (a) $2x + 18$ (b) $5a + 2b$
   (c) $6x + 8y$ (d) $8a + 8$
   (e) $12x + 2$ (f) $2x + 6$
   (g) $30x^3 - 18x^2 - 42x$ (h) $24y^3 + 32y^2 - 72y$
   (i) $-8m^5 - 4m^4 + 24m^3$ (j) $-21h^6 + 15h^4 + 33h^3$

Practice Exercise 16

1. (a) $m^2 - 9$ (b) $x^2 - 3x - 10$
   (c) $y^2 + 4y - 21$ (d) $3m^2 - 5m + 2$

2.

$$\begin{array}{rrrr} & -2x^2 & + 4x & + 11 \\ & & 5x & + 12 \\ \hline & -24x^2 & + 48x & + 132 \\ -10x^3 & + 20x^2 & + 55x & \\ \hline -10x^3 & - 4x^2 & + 103x & + 132 \\ \hline \end{array}$$

3 $-11x\,(10x^2 - 7x + 9) + 3(10x^2 - 7x + 9)$
$= -110x^3 + 77x^2 - 99x + 30x^2 - 21x + 27$
$= -110x^3 + 107x^2 - 120x + 27$

4. (a) $a^5 + 3a^4 + 3a^3 + 7a^2 - 2a + 12$
   (b) $b^5 - 7b^4 + 10b^3 + 3b^2 - 20b + 25$

Practice Exercise 17

1. $9x^2 - 3$
2. $2a^5 - 1$
3. $27x^5 - 15$
4. $n - 3$
5. $4 + x$
6. $7a^3 + 1$
7. $6a^2 - 4$
8. $10y^{-1} - y^2$
9. 5
10. 4

Practice Exercise 18

1. (a) $27x^2 \div 9 = \dfrac{27x^2}{9} = 3x^2$

(b) $27x^2 \div (-3) = \dfrac{27x^2}{-3} = -9x^2$

(c) $40x^2 + 28) \div 4 = \dfrac{40x^2 + 28}{4} = \dfrac{40x^2}{4} + \dfrac{28}{4} = 10x^2 + 7$

(d) $(6p - 3) \div (-3) = \dfrac{6p - 3}{-3} = \dfrac{6p}{-3} - \dfrac{3}{-3} = -2p + 1$

(e) $(-12y^5 - 10x^3) \div (-2) = \dfrac{-12y^5 - 10x^2}{-2} = \dfrac{-12y^5}{-2} - \dfrac{10x^3}{-2} = 6y^5 + 5x^3$

(f) $(18a^2 - 12b^2 + 6) \div 6 = \dfrac{18a^2 - 12b^2 + 6}{6} = \dfrac{18a^2}{6} - \dfrac{12b^2}{6} + \dfrac{6}{6} = 3a^2 - 2b^2 + 1$

2. (a) $y^{10} \div y^8 = \dfrac{y^{10}}{y^8} = y^2$

(b) $x^9 \div (-x^3) = \dfrac{x^9}{-x^3} = -x^6$

(c) $20z^4 \div 10z = \dfrac{20z^4}{10z} = 2z^3$

(d) $2f^4 \div (-3f^2) = \dfrac{2f^4}{-3f^2} - \dfrac{2}{3}f^2$

(e) $\dfrac{4r^2 - 8}{2} = \dfrac{4r^2}{2} - \dfrac{8}{2} = 2r^2 - 4$

(f) $\dfrac{12r^4 - 3r^3}{r^3} = \dfrac{12r^4}{r^3} - \dfrac{3r^3}{r^3} = 12r - 3$

(g) $\dfrac{27r^4 - 12r^2 + 9r}{-3r} = \dfrac{27r^4}{-3r} - \dfrac{12r^2}{-3r} + \dfrac{9r}{-3r} = -9r^3 + 4r - 3$

END OF SUB STRAND 3

SUB - STRAND 4

MATHEMATICAL EQUATIONS

SUB- STRAND 4: MATHEMATICAL EQUATIONS

Introduction

Dear Student,

Welcome to Sub-strand 4 of your Strand 6.

Mathematical Equations make it easy to solve complicated mathematical problems.
Learning and mastering rules and laws of algebra makes it even easier to solve mathematical equations.

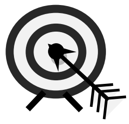
You must revise all you have studied in Sub strand 1 to 3 of this strand in order to do well in this Sub strand.

In this sub-strand you will:

- solve simple algebraic expressions by substitution
- apply problem-solving techniques to solve problems
- manipulate simple algebraic expressions and solve real life problems

Lesson 19: Mathematical Expressions and Sentences

You learnt something about mathematical expressions in Grade 7 Strand 6 Sub-strand 4.



In this lesson, you will:

- identify mathematical expressions and sentences.
- recognize mathematical sentences as true, false or open.
- find the solution set of a mathematical sentence.

Being able to comprehend mathematical expressions can help us solve many real life problems. You have studied algebraic expressions in sub-strand 1 of this strand. This lesson will be an extension of what you have learnt.

In sub-strand 1 we identified Mathematical expressions as a series of terms added, subtracted, multiplied or divided. In this lesson we will further identify Mathematical expressions as:

A combination of operations, constants and variables representing numbers or quantities.

Examples

1) $1 + 2$
2) $6 – (2 + 4)$
3) $\frac{1}{2}(5)$
4) $2x + 1$
5) $5x$

You translated or wrote phrases, expressions and stories mathematically in sub-strand 1 and the examples were,

1) Add one and five is written as $1 + 5$
2) The difference of 10 and a number is written as $10 – x$
3) The product of 5 and a number is written as $5n$
4) The quotient of 20 and 5 is written as $20 \div 5$

There were key terms you learnt in sub-strand 1 of this strand that meant an operation.

- Plus, together, total, put together, grouped are words that mean addition.
- Difference between, take away, decrease are words that mean subtraction.

Here are some more examples

Words	Meaning	Mathematical Expression
Twice a number	2 times a number	2x
Decrease a number by 5	A number minus 5	x - 5
Sum of a number and 3	Add a number to 3	x + 3
Increase 6 by a number	Add a number to 6	x + 6

What we have discussed so far are mathematical expressions. Mathematical sentences are slightly different to mathematical expressions.

Example

Mathematical Expressions	Mathematical Sentences
2	1 + 1 = 2
1 + 1	1 + 1 = 3
x + 1	x + 1 = 3

A mathematical *sentence* is a correct arrangement of mathematical symbols that states a complete thought. It can also be written out as a sentence in words.

Mathematical sentences can be judged as true or false.

Examples

1) 1 + 1 = 2 is a true sentence.
2) 1 + 1 = 3 is a false sentence. 1 + 1 = 2 not 3.
3) x + 1 = 3, is sometimes true and sometimes false depending on the value of the variable x.
4) An isosceles triangle has two congruent sides, is a true mathematical sentence.
5) 1 + 42 = 15 is a false mathematical sentence. 1 + 42 is 43 not 15.
6) x + 3 = 3 + x is always true no matter what number is chosen for x.
7) x = 2 , is sometimes true and sometimes false, It is true when x is 2.

There are two types of mathematical sentences. Sentences can be:

1) Open
2) Closed

In this lesson you will study Open sentences.

An open sentence contains a variable. The true value of this sentence depends up on the value replacing the variable.

Examples

1) A number plus ten is twenty one
2) $x + 10 = 21$
3) $25 – 2x = 15$
4) Two thirds of a number is 61.
5) Half of 100 is a number.

In summary a mathematical sentence is an arrangement of variables, operational signs and symbols showing a complete thought or idea. The mathematical sentence becomes an open sentence when it contains a variable whose value is unknown. The true value of the sentence depends on the value replacing the variable.

The value replacing the variable which makes the sentence true is called the solution set.

Examples

1. A number plus ten is twenty one, this sentence is true when the variable" a number" is replaced by eleven. Therefore we can say the solution is eleven.
2. $x + 10 = 21$, this sentence is true when the variable" x" is replaced by 11. Therefore we can say the solution set is 11.
3. $25 – 2x = 15$, this sentence is true when $x = 5$, we therefore say that the solution is 5.
4. A number is less than five, as a mathematical sentence, it can be written as $x < 5$. The solution set is 1, 2, 3, 4 .

NOW DO PRACTICE EXERCISE 19

Practice Exercise 19

1. Find the meaning of the following words

   (a) Mathematical expression

   (b) Mathematical sentence

   (c) Open sentence

---

2. Classify the following mathematical sentences as true, false or open.

   (a) x + 1 = 0

   (b) A square has 4 congruent sides.

   (c) 1 + 4 = 4

   (d) A trapezoid is a 4 sided polygon.

---

3. Find the solution set for the following sentences.

   (a) x – 9 = 21

   (b) a number less than 10 and greater then 5

   (c) multiples of 2 less than 8.

---

4. What are some other ways we can change the mathematical sentence 4 + 3 =7 in to an algebraic sentence?

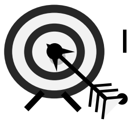
   Answer:
   (a) 4 **plus** 3 is 7
   (b) **sum** of 4 and 3 is 7
   (c) 3 **added to** 4 is 7
   (d) 4 **increased by** 3 is 7
   (e) 3 **more than** 4 is 7

   4 + 3 = 7 was done as an example. Now translate 8 – 6 = 2.

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 4

Lesson 20: Solving Equations

You have translated sentences to mathematical expressions and vice versa in the last lesson.



In this lesson, you will:

- identify the three parts of an equation.
- define the root or the solution of an equation.
- identify the principles of equality.
- solve simple equations using the principles of equality.

In the previous lesson, you classified mathematical sentences as open, true or false as well as finding the solution set for an open mathematical sentence. This lesson will help you extend your knowledge of mathematical sentences and solution sets of open sentences called **EQUATIONS**.

An equation is a mathematical sentence or statement that two expressions are equal. The two expressions are separated by an equal sign (=). These make up the three parts of an equation.

Examples

1) $2x + 4 = 12$
2) $x - 3 = 2$
3) $2x = 0.5$

The **solution** sometimes called the **root** of an equation is the value that when substituted for the variable makes the equation a true statement.

The main aim in solving an equation is to separate the variable on one side of the equation and the number on the other side of the equation so the equation will read **variable = number**.

The three examples of equations above have not been solved, once the equations are solved then they will be in the form **variable = number**, where the number is called the **solution** or **the root** of the equation. The following are the roots of examples 1 and 2.

Equation	Solution or root
$2x + 4 = 12$	$x = 4$
$x - 3 = 2$	$x = 5$

To solve an equation, you must learn the Principles or Properties of Equality. The principles or properties of equality state that whatever you do to one side of an equation must be done to the other side to keep the value equal.

Here are the Principles or Properties of Equality.

The Addition and Subtraction Principle of Equality

- **If the same number is added or subtracted to both sides of an equation the results on both sides are equal in value.**
- **If a number is added to a variable, subtract this number from both sides of the equation.**
- **If a number is subtracted from a variable, add this number to both sides of the equation.**

The Multiplication and Division Principle

- **If an equation is multiplied or divided by a nonzero number the result on both sides is equal in value**.
- **If a variable is multiplied by a number divide both sides of the equation by that number**.
- **If the variable is divided by a number multiply both sides of the equation by that number**.

To solve an equation you must follow the following steps.

Step 1: Remove parentheses by the distributive law (if any)

Step 2: Combine any like terms found on the same side.

Step 3: Use the additive principle to move the variable term to one side of the equation and the number to the other side.

Step 4: Use the multiplication or division principle to solve for the variable.

Step 5: Check the result in the original equation.

Examples

1) Solve 2x + 4 = 12

Solution: $2x + 4 = 12$ Addition principle

$$2x + 4 - 4 = 12 - 4$$

$$2x = 8$$ Multiplication principle

$$\frac{2x}{2} = \frac{8}{2}$$

$$\mathbf{x = 4}$$

Check by substituting the solution into the original equation, if it satisfies the equation then the solution is correct.

$$2x + 4 = 12$$
$$2(4) + 4 = 12$$
$$12 = 12$$ i.e. LHS = RHS

The left side of the equation is equal to the right side therefore It satisfies the equation and so the solution x = 4 is correct.

2) Solve $x - 3 = 2$

Solution: $x - 3 = 2$ Addition principle

$x + 3 - 3 = 2 + 3$

$x = 5$ Answer

Check: $x - 3 = 2$

$5 - 3 = 2$

$2 = 2$

3) Solve $2x = 0.5$

Solution: $2x = 0.5$ Multiplication principle

$\frac{2x}{2} = \frac{0.5}{2}$

$x = 0.25 = \frac{1}{4}$ Answer

Check: $2x = 0.5$

$2 \times 0.25 = 0.5$

$0.5 = 0.5$

4) Solve: $x + 5 = -7$

Solution: $x + 5 = -7$ Addition principle

$x + 5 - 5 = -7 - 5$

$x = -12$ Answer

5) Solve: $7x = 77$

Solution: $7x = 77$ Multiplication principle

$\frac{7x}{7} = \frac{77}{7}$

$x = 11$ Answer

6) Solve: $\frac{x}{5} = 2$

Solution: $\frac{x}{5} = 2$ Multiplication principle

$\frac{x}{5} \times 5 = 2 \times 5$

$x = 10$ Answer

NOW DO PRACTICE EXERCISE 20

Practice Exercise 20

1. Solve these equations using the Additive principle.

(a) $a + 6 = 12$ (b) $b - 5 = 14$ (c) $y + 3 = 9$

2. Solve these equations using the multiplicative principle.

(a) $\frac{x}{7} = 2$ (b) $\frac{m}{10} = 2$ (c) $\frac{t}{12} = 6$

(d) $8n = 56$ (e) $4p = 12$ (f) $2b = 88$

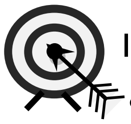
3. What is the solution to $9x + 3x = 132$?

4. Is (-4) the root to the equation $-3x - 5x = 32$?

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 4

Lesson 21: Equations with Pronumerals on Both Sides

You learnt to work out the solution or root of an equation using the principles of equality in the previous lesson.



In this lesson, you will:

- solve equations with pronumerals on both sides using addition and subtraction.

You have learnt to work out equations whose unknown quantity (pronumeral or variable) is found only on the left side of the equal sign. In this lesson, we will work out the solution to equations with pronumerals or variables on both sides.

These are examples of equations with pronumerals or variables on both sides.

1) $9x + 6 = 8x + 9$
2) $5 - 3x = 2x + 3$
3) $3a + 5 = 2a + 7$
4) $5x - 3 = 2x + 9$

You will be required to review the principles of equality so as to solve equations with pronumerals on both sides of an equation.

When a pronumeral or variable is on both sides of an equation, remove the pronumeral term from the right hand side of the equation by using the principles of equality. Then continue to solve the equation for the pronumeral using the steps learnt in Lesson 20.

Here are examples on how to solve these kinds of equations. In each example each important step is **bolded** and the principles used are shown to the right side. Follow carefully each example. It is easy to follow if you have remembered the rules in algebra and the steps to solving equations as well as the principle of equality.

Example 1

$9x + 6 = 8x + 9$ Addition principle to move 8x

$9x \mathbf{- 8x} + 6 = 8x \mathbf{- 8x} + 9$

$x + 6 = 9$ Addition principle to move 6

$x + 6 \mathbf{- 6} = 9 \mathbf{- 6}$

$\mathbf{x = 3}$

Check: $9x + 6 = 8x + 9$

$9(3) + 6 = 8(3) + 9$

$33 = 33$

Both sides are equal, therefore the solution x = 3 is correct.

Example 2

$5 - 3x = 2x + 3$	Addition principle to move 2x
$5 - 3x - 2x = 2x - 2x + 3$	
$5 - 5x = 3$	Addition principle to move 5
$5 - 5 - 5x = 3 - 5$	
$-5x = -2$	Multiplication principle to move -5
$\frac{-5x}{-5} = \frac{-2}{-5}$	
$\mathbf{x = \frac{2}{5}}$	**Answer**

Check: $5 - 3x = 2x + 3$

$$5 - 3\left(\frac{2}{5}\right) = 2\left(\frac{2}{5}\right) + 3$$

$$5 - \frac{6}{5} = \frac{4}{5} + 3$$

$$\frac{19}{5} = \frac{19}{5}$$

The Solution $\frac{2}{5}$ is correct since both sides are equal.

Example 3

$3a + 5 = 2a + 7$	Addition principle to move 2a
$3a \mathbf{- 2a} + 5 = 2a \mathbf{- 2a} + 7$	
$a + 5 = 7$	Addition principle to move 5
$a + 5 \mathbf{- 5} = 7 \mathbf{- 5}$	
$\mathbf{a = 2}$	**Answer**

Check: $3a + 5 = 2a + 7$

$$3(2) + 5 = 2(2) + 7$$

$$6 + 5 = 4 + 7$$

$$11 = 11$$

The solution a = 2 is correct since both sides are equal.

Example 4

$5x - 3 = 2x + 9$	Addition principle to move 2x
$5x \mathbf{- 2x} - 3 = 2x \mathbf{- 2x} + 9$	
$3x - 3 = 9$	Addition principle to move 3
$3x - 3 \mathbf{+ 3} = 9 \mathbf{+ 3}$	
$3x = 12$	Multiplication principle to move 3
$\frac{3x}{3} = \frac{12}{3}$	
$\mathbf{x = 4}$	**Answer**

Check:

$$\begin{gathered} 5x - 3 = 2x + 9 \\ 5(4) - 3 = 2(4) + 9 \\ 20 - 3 = 8 + 9 \\ 17 = 17 \end{gathered}$$

The solution x = 4 is correct since both sides are equal..

Example 5

$2 - 2x = 8 + x$	Addition principle, move x
$2 - 2x \mathbf{- x} = 8 + x \mathbf{- x}$	
$2 - 3x = 8$	
$2 \mathbf{- 2} - 3x = 8 \mathbf{- 2}$	Addition principle, move 2
$-3x = 6$	Multiplication principle, move -3
$\frac{-3x}{-3} = \frac{6}{-3}$	
$\mathbf{x = -2}$	**Answer**

The process in solving equations with pronumerals on both sides of an equation is just like solving any other simple equation.

NOW DO PRACTICE EXERCISE 21

Practice Exercise 21

1. Solve the following equations

(a) $2a + 5 = a + 7$ (b) $3x + 1 = 2x + 8$

(c) $7p - 2 = 6p + 1$ (d) $10q - 3 = 9q + 5$

2. Check to see if the given solution for each equation is correct.

(a) $5m - 2 = 4m + 1$ $m = 3$

(b) $3p + 7 = 2p + 12$ $p = 5$

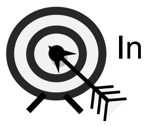
3. Solve these equations

(a) $5a - 1 = 3a + 4$ (b) $12n - 9 = 9n + 1$ (c) $8q - 5 = 5q + 3$

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUB STRAND 4

Lesson 22: Equations with Grouping Symbols

In the previous lesson, you learnt how to solve equations involving variables on both sides of the equation.



In this lesson, you will:

- solve equations with grouping symbols.

In Lesson 20 and 21, you learnt how to solve equations with variables on one side of the equation and variables on both sides of the equation. In both processes you were required to use the Principle of Equality to achieve the goal of finding solutions.

Equations with grouping symbols are solved by:
- **Removing the grouping symbol by the distributive law.**
- **Then solving the equation by using the steps learnt in Lesson 20 and the principle of equality.**

Before you study examples on solving equations, let us review the distributive law which you studied in sub-strand 1 of this strand. The distributive law states that:

$$a(b + c) = ab + ac$$

$$a(b - c) = ab - ac$$

Now study the following examples of solving equations with grouping symbols. Some steps are **bolded** to show the importance of rules or laws used. The rule at each step is shown below on the right of your workbook.

Example 1

$2(x + 4) = 10$ distributive law, remove grouping symbol.

$2x + 8 = 10$ addition principle, move 8

$2x + 8 \mathbf{- 8} = 10 \mathbf{- 8}$

$\mathbf{2}x = 2$ multiplication principle, move 2

$$\frac{2x}{2} = \frac{2}{2}$$

$$x = 1$$

Check:

$$2(x + 4) = 10$$

$$2(1 + 4) = 10$$

$$2(5) = 10$$

$$10 = 10$$

The solution x = 1 is correct because the LHS = RHS.

Example 2

$5(a - 3) = 3$	Distributive law
$5a - 15 = 3$	Addition principle move 15
$5a - 15 \mathbf{+ 15} = 3 \mathbf{+ 15}$	
$\mathbf{5}a = 18$	Multiplication principle, move 5
$\frac{5a}{5} = \frac{18}{5}$	
$a = \frac{18}{5} = \mathbf{3\frac{3}{5}}$	**Answer**

Example 3

$3(2m - 4) = 4m - 6$	Distributive law
$6m - 12 = 4m - 6$	Addition principle move 4m
$6m \mathbf{- 4m} - 12 = 4m \mathbf{- 4m} - 6$	
$2m - 12 = -6$	
$2m - 12 \mathbf{+ 12} = -6 \mathbf{+ 12}$	Addition principle, move 12
$2m = 6$	Multiplication principle, move 2
$\frac{2m}{2} = \frac{6}{2}$	
$\mathbf{m = 3}$	**Answer**

Example 4

$3(a + 7) = 4(a - 2)$	Distributive law
$3a + 21 = 4a - 8$	Addition principle, move 4a
$3a \mathbf{- 4a} + 21 = 4a \mathbf{- 4a} - 8$	
$-a + 21 = -8$	Addition principle, move 21
$-a + 21 \mathbf{- 21} = -8 \mathbf{- 21}$	
$-a = -29$	Multiply both sides by -1
$\mathbf{a = 29}$	**Answer**

Note that the last step in solving the equation in example 4, required us to multiply by -1, this is because in solving a equation a variable must always be positive therefore by the identity law when the negative variable is multiplied by -1 gives a positive variable.

Example 5

$3(x + 4) + 2(x - 5) = 4$	Distributive law, remove parentheses
$3x + 12 + 2x - 10 = 4$	Combine like terms
$3x + 2x + 12 - 10 = 4$	
$5x + 2 = 4$	Addition principle, move 2
$5x + 2 \mathbf{- 2} = 4 \mathbf{- 2}$	
$5x = 2$	Multiplication principle move 5
$\frac{5x}{5} = \frac{2}{5}$	
$\mathbf{x = \frac{2}{5}}$	**Answer**

Example 6

Solve $5(4t - 14) - 7 = 63$

Solution:	$5(4t - 14) - 7 = 63$	Distributive law
	$20t - 70 - 7 = 63$	Combine like terms
	$20t - 77 = 63$	Addition principle
	$20t - 77 \mathbf{+ 77} = 63 + \mathbf{77}$	
	$\mathbf{20}t = 140$	Multiplication principle
	$\frac{20t}{20} = \frac{140}{20}$	
	$\mathbf{t = 7}$	**Answer**

We have not checked solutions to example 2 to 6; you can do that yourself to see if the solutions are correct.

NOW DO PRACTICE EXERCISE 22

Practice Exercise 22

1. Expand the grouping symbols and then solve each equation.

   (a) $2(x + 3) = 4$

   (b) $3(a + 2) = 12$

   (c) $7(n + 2) = 14$

   (d) $6(p + 2) = 24$

2. Solve each equation

   (a) $4(n - 2) = 2n + 4$

   (b) $3(2a + 1) = 4a + 7$

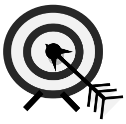
   (c) $2(m + 5) = 3(m - 10)$

   (d) $6x - 3 = 3(x + 2)$

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUBTRAND 4

Lesson 23: Formulas

You learnt to solve equations using the different properties of equality in the previous lessons.



In this lesson, you will:

- describe what a formula is
- derive a formula from another formula
- evaluate a formula.

In the last three lessons you learnt how to solve equations. The rules and steps you learnt in solving equations are also useful in this lesson.

A formula is made up of numbers and symbols that show you how to work something out. It is a special type of equation that shows the relationship between different variables.

Formula is singular and Formulae or formulas are plural.

These are some examples of formulas that are used in mathematics today.

Formula	Variables of the formula
A = b x h	This formula is for finding the area of a rectangle. **A** stands for area, **b** for the base and **h** for the height.
V = w x d x h	This formula is for finding the volume of a box. **V** stands for volume, **w** for width, **d** for depth and **h** for the height.
S = d x t	This formula is for finding speed. **S** stands for speed, **d** for distance and **t** for time.
P = 2l + 2w	This is the formula for finding the perimeter of a rectangle. **P** stands for perimeter, **l** for length and **w** for width.
$A = \frac{1}{2}(a+b)h$	This is the formula for finding the area of a trapezium. **A** stands for area, **a** and **b** are the parallel sides and **h** is the height.

A formula is an equation which specifies how a number of variables are related to one another. Formulas are written so that a single variable, the **subject** of the formula, is on the left hand side of the equation. Everything else goes on the right hand side of the equation.

Example In the formula **S = dt**, **S** is the subject of the formula.

Formulas are used to calculate the value of the subject when all the values of the other variables on the right-hand side are known. To find **S** in the example you must substitute the values of **d** and **t** to give the value of S.

To find the values of the other variables like **d** and **t**, you can make any of these other variables the subject instead of **S**.

> **1) To change the subject of the formula, identify the variable to become the new subject then use the principle of equality and the steps to solving equations to move the subject to the left-side of the equation and the other variables to the right side.**
>
> **2) To evaluate, substitute the values of the known variables and simplify.**

Example 1 Given **S = dt**, make **t** the subject of the equation.

Solution: $S = dt$ Multiplication principle move d.

$\frac{S}{d} = \frac{dt}{d}$

$\frac{S}{d} = t$ Move subject to the left side.

$\mathbf{t = \frac{S}{d}}$

t is now the subject of the equation.

Example 2 Given **V = u + at**

a) Make **u** the subject of the equation.

b) Find the value of **u** when **V** = 5, **a** = 2 and **t** = 1.

Solutions: (a) $V = u + at$ Addition principle, move **at**

$V – at = u$ Move subject to the left side

u = V – at **Answer**

(b) From (a) you can find **u** when **V** = 5, **a** = 2 and **t** = 1

This can be solved by substituting the known values of the variables into **u = V – at.**

$u = V – at$

$u = 5 – 2(1)$

$u = 5 – 2$

u = 3 **Answer**

Example 3 Given **A = bh**,

a) Express **h** in terms of **A** and **b**.

b) If A = 20 and b = 4, find the value of h.

Solutions: (a) $A = bh$ Multiplication principle move b.

$$\frac{A}{b} = \frac{bh}{b}$$

$$\frac{A}{b} = h$$ Write h on the left side.

$$\mathbf{h = \frac{A}{b}}$$

(b) By substitution

$$h = \frac{A}{b}$$

$h = \frac{20}{4} = 5$, thus **$h = 5$**

Example 4 Given P = 2l + 2w, express **w** in terms of **P** and **l**.

Solution: $P = 2l + 2w$ Additive principle, move 2l.

$$P - \mathbf{2l} = 2l - \mathbf{2l} + 2w$$

$P - 2l = \mathbf{2}w$ Multiplicative principle, move 2.

$$\frac{P - 2l}{2} = \frac{2w}{2}$$

$\frac{P - 2l}{2} = w$ or $w = \frac{P - 2l}{2}$ Move **w** to left side.

Example 5 Given V = wdh; (a) Make **h** the subject.

(b) Find h when V = 240, w = 10 and d = 5.

Solutions: a) $V = wdh$

$V = wdh$ Multiplicative principle move **wd**

$$\frac{V}{wd} = \frac{wdh}{wd}$$

$\frac{V}{wd} = h$ Move **h** to the left side

$$h = \frac{V}{wd}$$

b) By substitution, $h = \frac{V}{wd} = \frac{240}{10 \times 5} = \frac{240}{50} = 4.8$, therefore **$h = 4.8$**

NOW DO PRACTICE EXERCISE 23

Practice Exercise 23

1. Given v = u + at.

   a) Make **u** the subject.

   b) Find **u** when v = 20, a = 5 and t = 2

2. P = 4s is the formula for finding the perimeter of a square. Write **s** in terms of P.

3. Evaluate the following formulas. Use the given values to find the values of the remaining variables.

   (a) $F = \frac{9c}{5} + 32$ c = -20 find F

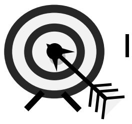
   (b) $A = \frac{x + y + z}{3}$ A = 6, x = 9 and y = 2 find z

4. Given P = 2l + 2w make l the subject of the equation.

CORRECT YOUR WORK.ANSWERS ARE AT THE END OF SUB-STRAND 4

Lesson 24: Solving Word Problems Involving Equations in One Unknown

In the previous lessons we learnt how to solve equations. You must become familiar with the steps in solving equations so you may apply it to this lesson.



In this lesson, you will:

- solve worded problems using equation-solving techniques.

The main use of algebra, and particularly equations, is to help us solve problems. You have learnt how to translate words and phrases to mathematical expressions and sentences. Examples of some of the words you learnt are presented in the table below.

Addition	Subtraction	Multiplication	Division	Equals
Sum Add Put together Altogether Increased by More than	Take away Subtract Decreased by Difference of Less than Difference between Less than Fewer than	Product of Of Times Twice Multiplied by	Per Out of Quotient Divide by Share Ratio of	Is Are Was Were Will be Gives Yields

You can revise Sub-strand 1 on the mathematical phrases. These phrases are important in solving worded problems. Remembering the phrases and what they mean can help you construct and solve an equation as you have learnt in Sub-strand 1.

Many of the problems in this sub-strand are very easy. You may be able to see the answers immediately. But you must practice writing out solutions correctly in order to be able to solve harder equations and problems later.

Follow these steps to solve a worded problem.

Step 1: Read the problem carefully and find out what is being given and what is being asked.

Step 2: Represent the unknown or what you are looking for with a variable.

Step 3: Translate the problem into an algebraic sentence or an equation.

Step 4: Solve the equation.

Step 5: Check.

Example 1

The sum of a certain number and 7 is 12. What is the number?

Solution:

Step 1: Let the certain number be n

Step 2: $n + 7 = 12$

Step 3: $n + 7 – 7 = 12 – 7$

$n = 5$

The number is 5.

Example 2

The product of a number and 9 is 72. What is the number?

Solution:

Step 1: Let the number be x

Step 2: $9x = 72$

Step 3: $\frac{9x}{9} = \frac{72}{9}$

$x = 8$

The number is 8.

Example 3

After buying ice block, I have 35t left from K1.00. How much was the ice block?

Solution: Let the cost of the ice block be c toea

$c + 35 = 100$

$c + 35 – 35 = 100 – 35$

$c = 65$

The cost of the ice block was 65 toea.

Example 4

If an integer is multiplied by 7, and 3 is added to the product, the result is 38. What is the integer?

Solution: Let the integer be x.

$$7x + 3 = 38$$

$$7x + 3 - 3 = 38 - 3$$

$$7x = 35$$

$$\frac{7x}{7} = \frac{35}{7}$$

$$x = 5$$

The integer is 5.

In the next two examples you have to collect like terms before solving the equation.

Example 5

One number is 5 more than a second number. Their sum is 21. What are the numbers?

Let the one number be n. The other number is n + 5 (one number is 5 more).

$$n + n + 5 = 21$$

$$2n + 5 = 21$$ Collect like terms

$$2n + 5 - 5 = 21 - 5$$

$$2n = 16$$

$$\frac{2n}{2} = \frac{16}{2}$$

$$n = 8$$

Since the second number is n + 5 then n + 5 = 8 + 5 = 13.

Therefore, the two numbers are 8 and 13.

Example 6

The perimeter of a triangle is 36 cm. The sides are 2x, 3x and 4x. Find the length of each side.

Solution: We know that the perimeter of a triangle equals the sum of its sides.

Since the perimeter is 36 and the sides are given as 2x, 3x and 4x,

we have $2x + 3x + 4x = 36$ as the equation

$9x = 36$ collect like terms

$x = 4$ solve by dividing both sides by 9

Since the sides are 2x, 3x and 4x, we have

$2x = 2 \times 4 = 8$ cm, $3x = 3 \times 4 = 12$ cm, $4x = 4 \times 4 = 16$ cm

Therefore, the side lengths are 8 cm, 12 cm and 16 cm.

Example 7

A number is added to 85 and the result is 172. What is the number?

Solution: Let the number be x

$$x + 85 = 172$$

$$x - 85 + 85 = 172 - 85$$

$$x = 87$$

Therefore, the number is 87.

Remember to solve a problem

- **Carefully read the problem.**
- **Assume that the unknown is *x* or another suitable variable.**
- **Write an equation using the information provided in the problem.**
- **Solve the equation by using the rules and steps you learnt in lessons 20 and 21.**
- **Write your answer in words.**

NOW DO PRACTICE EXERCISE 24

Practice Exercise 24

1. Solve these problems by first writing an equation.

   (a) The sum of a certain number and 4 is 13. What is the number?

   (b) The sum of 9 and a certain number is 17. What is the number?

   (c) A certain number minus 7 is equal to 9. What is the number?

   (d) The product of 8 and a number is 56. what is the number?

   (e) The total of a number, 6 and 14 is 35. What is the number?

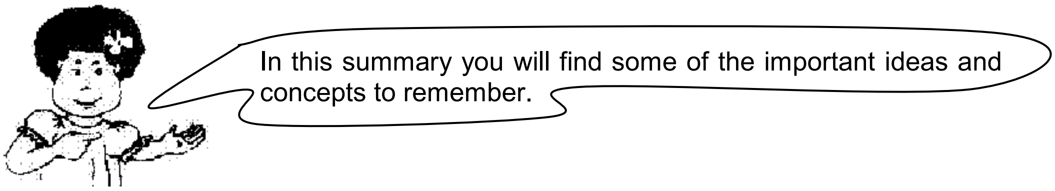
   (f) If a number is divided by 7, the result is 12. What is the number?

   (g) If a number is multiplied by 3, and 5 is added to that product, the answer is 17. What is the number?

   (h) If 10 is subtracted from the product of 2 and a certain number, the result is 4. What is the number?

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUB STRAND 4

SUB-STRAND 4: SUMMARY



- A mathematical expression is a series of terms added, subtracted, multiplied or divided. These expressions are also called algebraic expressions.
- An equation is a number sentence. It has three parts, an expression on the left side, an equal sign in the middle and an expression on the right-hand side.
- Solving an equation means to find the value of the unknown in an equation.
- Equations with an unknown on both sides can be solved by following the following steps:

  Step 1: Remove parentheses by the distributive law (if any)

  Step 2: Combine any like terms found on the same side.

  Step 3: Use the additive principle to move the variable term to one side of the equation and the number to the other side.

  Step 4: Use the multiplication or division principle to solve for the variable.

  Step 5: Check the result by substituting the computed value into the original equation

- Equations with grouping symbols must be removed by the distributive law before solving.
- A Formula is an instruction or rule usually in the form of an equation.
- Deriving a formula from a given formula can be done by the same process as solving equations.
- In order to solve equations easily you must master all the algebraic rules and laws covered in sub-strand 1 to 3 of strand 6.
- In order to do well, you must practice the same exercises often.

REVISE LESSON 19-24. THEN DO SUB-STRAND TEST 4 IN ASSIGNMENT 6.

ANSWERS TO PRACTICE EXERCISE 19 – 24

Practice Exercise 19

1.
   (a) Mathematical expressions consists of numbers, variables(terms) separated by plus or minus signs.
   (b) Mathematical sentence is a complete thought or idea. Sentences can judged as true or false.
   (c) Open sentences contain unknown variables.

2. a) Open b) true c) false d) true

3. a) 30 b) 6,7,8,9 c) 2,4,6

4. 8 minus 6 is 2
   8 decreased by 6 is 2.
   6 less than 8 is 2.
   Difference of 8 and 6 is 2
   8 reduced by 6 is 2.
   6 subtracted from 8 is 2

Practice Exercise 20

1. (a) $a = 6$ (b) $b = 19$ (c) $y = 6$

2. (a) $x = 14$ (b) $m = 20$ (c) $t = 72$
   (d) $n = 7$ (e) $p = 3$ (f) $b = 44$

3. $12x = 132$
   $x = 11$

Practice Exercise 21

1. (a) $a = 2$ (b) $x = 7$
   (c) $p = 3$ (d) $q = 8$

2. (a) $5m – 2 = 4m + 1$
   $5(3) – 2 = 4(3) + 1$
   $13 = 13$, the RHS = LHS, therefore the solution is correct.

   (b) $3p + 7 = 2p + 12$
   $3(5) + 7 = 2(5) + 12$
   $22 = 22$, the RHS = LHS, therefore the solution is correct.

3. (a) $a = \frac{5}{2}$ (b) $n = \frac{10}{3}$ (c) $q = \frac{8}{3}$

Practice Exercise 22

1. (a)

$2(x + 3) = 4$	remove grouping symbol
$2x + 6 = 4$	subtract 6 on both sides
$2x = -2$	divide 2 on both sides
$x = -1$	

(b)

$3(a + 2) = 12$	remove grouping symbol
$3a + 6 = 12$	subtract 6 on both sides
$3a = 6$	divide 3 on both sides
$a = 2$	

(c)

$7(n + 2) = 14$	remove grouping symbol
$7n + 14 = 14$	subtract 14 on both sides
$7n = 0$	divide 7 on both sides
$n = 0$	

(d)

$6(p+2) = 24$	remove grouping symbol
$6p + 12 = 24$	subtract 12 on both sides
$6p = 12$	divide 6 on both sides
$p = 2$	

2. (a)

$4(n – 2) = 2n + 4$	remove grouping symbol
$4n – 8 = 2n + 4$	combine like terms
$4n – 2n – 8 = 4$	
$2n = 4 + 8$	
$2n = 12$	divide by 2
$n = 6$	

(b)

$3(2a + 1) = 4a + 7$	remove grouping symbol
$6a + 3 = 4a + 7$	combine like terms
$2a = 4$	divide by 2
$a = 2$	

(c)

$2(m + 5) = 3(m – 10)$	remove grouping symbol
$2m + 10 = 3m – 30$	combine like terms
$-m = -40$	divide by -1
$m = 40$	

(d) $6x - 3 = 3(x + 2)$ remove grouping symbol

$6x - 3 = 3x + 6$ combine like terms

$3x = 9$ divide by 3

$x = 3$

Practice Exercise 23

1. (a) $u = V - at$ (b) $u = 10$
2. $s = \frac{p}{4}$
3. (a) $F = 68$

   (b) $Z = 7$
4. $l = \frac{P}{2} - w$

Practice Exercise 24

1. (a) $x = 9$ (b) $x = 8$ (c) $x = 16$

   (d) $x = 7$ (e) $x = 15$ (f) $x = 84$

   (g) $x = 4$ (h) $x = 7$

END OF SUB-STRAND 6

NOW YOU MUST COMPLETE ASSIGNMENT 6.
RETURN IT TO THE PROVINCIAL COORDINATOR.

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